

2014

## Teaching for students' confident transition from number to algebra

Christina Lang Lee  
*Edith Cowan University*

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Teaching for students' confident  
transition from number to algebra

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TEACHING FOR STUDENTS' CONFIDENT TRANSITION FROM NUMBER TO

ALGEBRA

CHRISTINA LANG LEE

Dip Ed

B Ed

M Ed

Submitted in fulfillment of the requirements of  
the Doctor of Philosophy in the  
Faculty of Arts and Education  
at Edith Cowan University

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## ABSTRACT

The adoption by all states and territories of the national curriculum by 2013 saw students in schools across the country taught introductory algebraic concepts from Year Five. In the twenty first century the need to be algebraically competent has become a necessity much as computation was in the previous centuries.

The Researcher has found from experience that students who have struggled with number and number operations will then most probably make poor progress in their study of algebra. The transition from number to algebra requires a robust understanding of number and number operations

This study investigated the balance of instructional strategies employed by teachers to support students transition from number to algebra. This research examined how teachers' beliefs underpinned the way that they approached the teaching of algebra in Years Seven and Eight in Western Australian schools.

This was a mixed methods study. The quantitative data from two questionnaires were used to analyse the teachers professed beliefs and also to contribute to the findings from the qualitative data to form the case study. The qualitative data was gathered from interviews, a focus group meeting, personal reflections by the teachers and video of four lessons taken of each teacher's practice. The greatest variation in content planning and teaching evident in the teachers' work was in the transitional material, namely moving students from working with number to algebraic variables.

This research makes a contribution to our (and teachers') knowledge about teaching algebra and it provides insights into understanding good practice in the teaching of beginning algebra.

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# Introduction

This chapter outlines the context, identifies the problem to be considered and the rationale and the purpose of the study, explains its significance, and concludes by listing the research questions.

## 1.1 Context

The Australian Curriculum in mathematics makes no explicit mention of the need to study pre algebra to progress to algebra, unlike the now defunct West Australian Curriculum Framework (Curriculum Council, 1998). Furthermore, the new national curriculum for mathematics (Australian Curriculum, Assessment and Reporting Authority (ACARA), 2013) links number and algebra under one strand (ACARA, 2013). In the document patterns are introduced from the kindergarten years and worked on throughout the primary years. Year Five is the point in the Australian Curriculum at which algebraic thinking is formally introduced in the curriculum, with students being taught to expand their idea of pattern to use tables, graphs and devise rules. Year Six sees the introduction of negative numbers. In Year Seven students are taught the concept of variables and how to use them. This is a major change for many Western Australian schools where algebraic thinking, integers and use of variables has traditionally been introduced in Year Eight.

The Australian Curriculum, Assessment and Reporting Authority (ACARA, 2009) reports that not enough students have been engaging in a positive way in algebraic thinking in the middle years of schooling; and, it would appear as a result that fewer students are choosing the more demanding mathematics courses in senior school which require the ability to use algebraic techniques at their core. The writers suggest possible impediments to student learning.

The personal and community advantages of successful mathematics learning can only be realized through successful participation and engagement. Although there are challenges at all years of schooling, participation is most at threat in Years 6—9. Student disengagement at these years could be

attributed to the nature of the curriculum, missed opportunities in earlier years, inappropriate learning and teaching processes, and perhaps the students' stages of physical development. (p. 9)

The Researcher contends that the movement of more formal early algebra to the primary curriculum will find many competent teachers feeling ill-equipped to deal adequately with this new curriculum.

## **1.2 The Problem**

Student achievement in mathematics at the Year Eight level internationally assessed in the Trends in International Mathematics and Science study shows that over one third of Australian Year Eight students failed to achieve the minimum proficient standard. The Year Eight students tested were weakest in algebra (TIMMS, 2011). Thomson, Hillman and Wernert, note that the 2012 TIMMS data show

- Nine per cent of Australian students achieved the Advanced international benchmark.
- A further 20 per cent achieved the High international benchmark.
- Thirty-seven percent of Australian students did not achieve the international benchmark, which is the minimum proficient standard expected.
- Year 8 students are weakest in algebra  
(pp. vi-vii).

The number of Year Eight student's failing to achieve the minimum standard of proficiency in algebra is a problem. Failure to engage in and learn the basics of algebra limits students' opportunities in senior school, tertiary study and ultimately in career choice.

### **1.3 The Rationale**

It is important to research the beliefs and practices of teachers to assess how these might make a difference to students' engagement with algebra, and their ability to work algebraically as they move into the middle years of schooling and an introduction to more formal algebra. In the past this has occurred not in Year Seven but in Year Eight classrooms in Western Australia.

Recent international test results indicate that Year Eight students in Australian schools as a cohort are weakest in algebra. In the twenty-first century there is a growing need in society for thinkers and problem solvers (Schoenfeld, 1995). Algebraic techniques foster higher-order thinking skills. If students want to study mathematics at the higher levels in senior school, an understanding of basic algebraic concepts is essential. The Australian Curriculum will see younger students in Western Australia exposed to using procedural algebraic techniques. Teachers who would not as a rule have taught algebra in the past will find themselves having to do so. It is in the early years of secondary school that students are introduced to the building blocks of formal algebra. The teaching of these foundational concepts is important and requires that teacher must be equipped to do so. Australia as a society cannot afford to squander the intellectual capacity of its young people. To take full advantage of the new job opportunities arising from advances made in science and technology both nationally and internationally students need to be at least at a basic standard in their knowledge and understanding of algebra.

### **1.4 Purpose and Research Questions**

The research contributes to the collective knowledge of the teaching of beginning algebra. This study explores the instructional strategies employed by four teachers in order to achieve success for learners. The teaching approaches used and developed by the four teachers are documented and analysed. This research examines how the four teachers' beliefs underpin the way they approach the teaching of algebra.



The research questions are as follows:

1. What were the teachers' beliefs about the teaching and learning of algebra?
  - (a) How did they view the educational importance of algebra?
  - (b) How did they consider the teaching of number?
  - (c) How did they choose to balance transmission, connectionist and discovery strategies to teach beginning algebra?
2. How does reflection upon these particular beliefs and practices inform the teachers' methods for fostering the successful conceptual development of key beginning algebraic ideas?

### **1.5 Significance**

The curriculum in other learning areas has changed substantially since the advent of schooling for all. Mathematics, at its core, still teaches the same principles, although some have changed as new knowledge has evolved or has been created or discovered.

The findings of this research will contribute to the existing knowledge of mathematics education by providing a useful insight into the teaching of beginning algebra and the beliefs of the four teachers that underpin their practice. Reflection on the four teachers' professed beliefs and their practice will also provide a useful lens from which to view future research into the efficacy of particular strategies chosen by teachers in terms of student learning outcomes in beginning algebra.

### **1.6 Overview of Study**

This research is informed by social constructivist and socio-cultural theory. The study examines how four teachers approached the teaching of beginning algebra and how their beliefs about mathematics influenced their practice.

# Literature Review

The review of the literature aims to provide a concise report of current research findings concerning the importance of algebra in the mathematics classroom, the cognitive processes and the difficulties in learning algebra, teacher beliefs about the teaching of early algebraic concepts, and teachers' knowledge of mathematics for teaching. Social constructivist and socio-cultural learning theories informed the study.

## 2.1 Background

Anna Sfard (2000b) writes that mathematics is a difficult subject and that mathematics teaching needs to be reformed.

Mathematics is one of the most difficult school subjects. It is therefore fully understandable why teaching mathematics has always been, and will probably always remain, subject to change and betterment. Improving the teaching of mathematics is the principal aim of the reform movement the presence of which can be felt these days practically all around the world. (Sfard, 2000b, p. 1)

## 2.2 The Importance of Algebra in the Mathematics Classroom

The Oxford English Dictionary defines 'algebra' as follows:

Algebra is the department of mathematics which investigates the relations and properties of numbers by means of general symbols; and, in a more abstract sense, a calculus of symbols combining according to certain defined laws. (Simpson & Weiner, 1989, p. 311)

The Researcher considers that the central words in this definition are *investigation*, *relations* and *properties of numbers*. Traditionally, education systems have viewed algebra as a strand of mathematics, which is taught in secondary school, and not necessarily to all students.

The paper Shape of the Australian Curriculum: Mathematics 2009, (National Curriculum Board, 2009), places number and algebra in the one strand. The paper cites the importance of introducing students to algebraic thinking long before all students begin secondary school. This movement to the introduction of algebraic thinking in the earlier years of schooling, particularly in relation to the structure and relationships in number, is commensurate with the changes taking place in curriculum planning in the USA (Kieran, 2007).

Recent research by Kaput (2008) and others would support this shift in curriculum planning and the claims that the integration of algebraic reasoning into everyday mathematics lessons is important from the earlier years (Britt & Irwin, 2006; Kaput, 2008; Mulligan, Cavanagh & Brown, 2012; Warren, 2003). According to Kaput, algebra is more than a continually developing body of knowledge, and is also a way of thinking, reasoning, talking and doing (Kaput, 2008). Viewed from this perspective, algebra includes both content and process. The content of algebra has its roots in cultural development. As society's needs have become more complex so too has the need to solve more abstract problems. Knowledge of the rules and conventions used in algebra is key to using algebraic techniques successfully to solve problems.

"Arithmetic in the elementary grades should be taught with a view to the algebraic thinking that students will need to develop as they progress through the curriculum" (Van Dooren, Verschaffel, & Onghena, 2002, p. 320). When students are introduced to the language of algebraic thought from an early age, they are more likely to engage in algebra lessons as the concepts become more demanding. Sfard (2000b) proposes that this initiation into the language of the mathematics community allows the learner to become more conversant with the common terms and conventions. When the algebraic language use becomes almost automatic, students are free to devote more time to contemplation of the construction of more abstract ideas. Sfard and Linchevski (1994) outline the historical development of algebraic language, linking the development of cognitive processes involved in learning school algebra to the historical development of algebra. The historical development of algebra identified by Sfard and Linchevski (1994) comprises three stages. The first was the rhetorical stage, attributed to Diophantus. This stage of development concentrated on the structure of algebraic representation. In the third century Diophantus introduced letters to represent *unknown* quantities, but these values were of a fixed nature; a letter could represent only one value in an expression. In a similar way, in schools, students are initially introduced to the

notion of a missing number in a calculation with a representation of a shape. There can be only one possible solution to this type of question. Thus in early algebra a letter is substituted for a missing value in an equation and there is only one possible solution. The second stage was to have far reaching consequences and is credited to the French mathematician Francois Viète (1540-1603). This was the beginning of the use of letters to stand for *variable* quantities. In school algebra when students understand that a letter can take the place of a number, they are then introduced to the concept of variable; the letter can now represent many different values. In the third stage, a more sophisticated notion than that of an unknown was used to model natural phenomena. Descartes combined the use of geometry and algebra to create the Cartesian Plane on which relationships between quantities could be represented and investigated. Fermat also contributed to this translation of algebra into the traditional field of geometry. When this construct had been developed, scientists such as Galileo and Newton could “represent natural phenomena and algebra was ultimately transformed from a science of constant quantities into a science of changing magnitudes” (Sfard & Linchevski, 1994, p. 200). For school mathematics this means that students are now taught that relationships can be represented using variables, which can change, and that these changes can be examined best using graphs.

The Researcher proposes that a reference to the historical development of algebra supports the thinking required to initiate students into the world of algebraic thought. Algebra has a purpose and the story of algebra shows how as society’s needs have increased so then has the idea of what algebraic thought can be to the participant.

As researchers have argued, students need to have a good understanding of and fluency in using the rules of arithmetic if they are to deal with the abstractions of algebra, because algebra initially grew out of operations with number. The New Zealand Numeracy Project (Britt & Irwin, 2005) found that when students could represent number operational strategies successfully they were more likely to be able to generalize, and therefore to illustrate algebraic thinking strategies. Fujii (2001) found that when students operated on numbers in a number sentence they demonstrated an understanding of the underlying mathematical relationship. He defined using numbers in this way as representing *quasi variable* thinking. Steffe (2001) argues that children’s knowledge of number together with operational knowledge of number that is effective and reliable is essentially algebraic in nature. Britt and Irwin suggest that there is no definable conclusion to the arithmetic in learner’s

minds, which precedes the beginnings of algebra. Dehaene (2011) proposes that the ability to identify pattern is innate. Few would disagree that pattern plays a large role in mathematics and in algebra. The Researcher has found over many years of experience as a teacher of mathematics that when the student begins to see the patterns the world of algebra becomes interesting and inviting.

Developing learning communities where all students have opportunities to engage in mathematical practices that support algebraic reasoning has been an increasing focus both in national and international research and curriculum reforms (Blanton & Kaput, 2003; National Council of Teachers of Mathematics, 2008). It is important that all students in their earlier years of schooling are exposed to the ideas and rules of algebra so that they may have the best opportunity to study mathematics at the highest level in the senior school. If mathematics is viewed as a flexible thinking subject (Boaler, 1998), then a mathematics classroom should give students many opportunities to develop their ability to think mathematically and to use their knowledge and understanding to solve problems in different situations. It is still sadly true that algebra in too many classrooms is taught as a series of rules and procedures, which have no connection to real life. Students who do not understand and do not learn the rules sufficiently well are limited in their choices of senior courses, and thus later in their choice of career (Kaput, 2000).

The RAND Corporation is a USA-based nonprofit institution whose aim is to help policy and decision-making through research and analysis. A 2003 report by the RAND Mathematics Study Panel identified the teaching and learning of algebra in kindergarten through to the 12<sup>th</sup> grade in its proposed research agenda. The rationale for the importance of algebra is offered in a threefold format, but in the Australian context it would be more pertinent to argue for just two of the points made in the report. The first argument is that:

Algebra is foundational in all areas of mathematics because it provides the tools (i.e., the language and structure) for representing and analyzing quantitative relationships, for modeling situations, for solving problems, and for stating and proving generalizations. (Ball, 2003 p. 44)

The second assertion is that algebra can be viewed as a “gatekeeper” course.

Without proficiency in algebra, students cannot access a full range of educational and career options, and this curtailment of opportunities often falls most directly on groups that are already disadvantaged. (Ball, 2003 p. 47)

Alan Schoenfeld (1995) also supports this idea:

There is a new literacy requirement for citizenship. Algebra today plays the role that reading and writing did in the industrial age. If one does not have algebra, one cannot understand much of science, statistics, business, or today's technology. Thus, algebra has become an academic passport for passage into virtually every avenue of the job market and every street of schooling. With too few exceptions, students who do not study algebra are therefore relegated to menial jobs and are unable often even to undertake training programs for jobs in which they might be interested. They are sorted out of the opportunities to become productive citizens of our society. (p. 11)

Algebra is about thinking and expressing this thinking in a particular way. Since the sixteenth century the use of symbolic algebra has developed and it continues to develop; the needs of our increasingly complex society and humanity's search for meaning and order will see the use of algebra essential for our world in the twenty first century (Bailey & Borwein, 2013).

## **2.3 Behaviourism**

In the classical traditions, thinkers were philosophers rather than psychologists or scientists. Plato (428-347B.C.) believed that knowledge was innate and that learning was merely the act of remembering. John Locke (1632-1704), however, developed a theory of learning which was to "profoundly influence the early development of modern psychology, as well as shape educational practice to the present day" (Phillips & Soltis, 2004, p. 13). Locke did not share Plato's belief that knowledge was innate, but believed that the human being is born with certain biological capacities but really knows nothing instinctively. He argued that it is only through experience that learning occurs.

It was not until the early twentieth century that theorists began to think about learning from a scientific approach. These theorists were not interested in the thinking behind the behaviour but rather in the behaviour itself. No one could observe or measure thinking but behaviours could be measured and compared (Bandura, 1963; Skinner, 1936; Thorndike, 1913; Watson, 1913). There are common threads within each of these theorists' writings about learning. The learner was essentially passive and responded to stimuli. Good or correct behaviour elicited a positive response while poor behaviour did not. The behaviourists believed that learning occurs with practice and immediate feedback.

Hartley (1998) identified four main beliefs about learning from a behaviourist perspective:

- Activity is essential.
- Repetition is important and practice should occur in many different contexts. This will assist the learner to discriminate and to generalize.
- Reinforcement is key. Correct behaviour is rewarded.
- Learning objectives are important and learning is enhanced when the objectives of the learning are made clear at the beginning of the lesson. Objectives can be measured and students given feedback on their learning. (pp.17-18)

From a behaviourist perspective the teacher is the source of information and the authority on knowledge in the classroom. The teacher's role is to transmit or pass on facts and processes. It is indeed from these theories that the transmission view of teaching and learning has been derived (Staub & Stern, 2002).

## **2.4 Social Constructivism and Socio-Cultural Theory**

Social constructivism and socio-cultural theories both propose that mathematics is a social construct. Socio-cultural theory proposes that the cultural context of the creation of the mathematics is central to the development of understanding in the learner (Vygotsky, 1986). Mathematics is as Ernest (1998) describes, "a web ... created that reflects the origin of mathematics as a language to describe, predict and regulate quantitative and special phenomena" (p. 263).

Piaget's mid twentieth-century theory of how children learn, centred on the notion that children construct their own knowledge from their life experiences. He investigated childhood development and as a result of his groundbreaking research important changes began to occur in classrooms around the world. In England and Wales the Plowden Report was released in 1967. The report emphasized the child at the centre of his or her learning, and it is claimed that was due to the work of Piaget on child development.

Chapter Two of the report, "The Children, their Growth and Development", is based firmly in Piagetian theory. This is hardly surprising as "during the 1960s this work by Piaget and his colleagues was at the peak of its influence. It was very widely known and very widely accepted" (Donaldson 1978, p. 34).

The emphasis in teaching moved from the body of knowledge that the teacher was to transmit, to the part the learner played, in his or her own learning. Piaget claimed that there were four distinct stages of development: sensorimotor, pre-operational, concrete operational and formal. In his theory individuals constructed meaning from an experience or interaction by themselves through the processes of assimilation, adaptation, accommodation and equilibrium. The learning or developmental pathway described by Piaget was essentially linear, and individuals could not move from one stage to another until they were ready. More research has been undertaken since the era of Piaget which suggests that the notion of a linear progression has flaws and that young children are capable of higher order thinking at lower levels and ages. Siegler (1998) proposes that while children typically reason much as Piaget describes they appear to have important cognitive capabilities that he did not detect. This research has implications for curriculum design and, as Dockett (1999) suggests, a young child is capable of working across previously identified stages of development when playing.

When the Researcher began teaching in the early 1970s, *discovery learning* was the major focus in curriculum design and classroom practice, something that had evolved from the Piagetian theory of learning. Polya (1965) is often described as the father of discovery learning due to his work concerning students' use of practical experience to solve problems and to build personal knowledge. In Polya's view, readiness for learning was crucial to the learning/teaching situation and the teacher's primary role was to provide materials and stimuli to encourage students to make meaning for themselves and to construct knowledge for themselves. The notion of discovery learning raised questions amongst teachers and educators



alike about the ability of young students to focus on what they were to discover. They were afraid that the tasks would be too open-ended. How could the teacher ensure that learning was happening? In the area of mathematics one answer proposed as a solution to this perceived problem was to instigate the use of investigations to allow students to discover an idea through the exploration of a problem by themselves.

In the Researcher's experience, the contrived nature of many of these types of investigations leaves little room for discovery and students can become frustrated and bored if they cannot see the pattern, or if there is no challenge. A more successful model to support student discovery is the use of inquiry, and in some forums "discovery learning" has been replaced by the term "inquiry learning". Discovery learning, as first proposed, failed to recognize the critical role of a child's prior knowledge which is needed to discover new relationships and understanding. In this thesis the Researcher takes the two terms to be related because inquiry learning is assumed to be a derivative of discovery learning, however, inquiry learning assumes that the child draws on relevant prior knowledge and skill to investigate and explore a new phenomena. In inquiry learning the scientific method is adapted for mathematics. The answer to a problem is discovered or achieved through a path of inquiry. Although similar to the practice of science, which formulates a hypothesis, in mathematics students need to clarify the problem to be studied and to choose suitable mathematics with which to work. The student then uses the strategy she has chosen and interprets her solution, making sure that she has answered the problem and that the answer makes sense (Polya, 1965).

Does learning happen only through personal experience or does learning happen through collaboration with others? There is a strong case for the argument that learning happens often as a group endeavor:

In later childhood and adolescence, children become increasingly able to collaborate effectively with each other. Such collaborations are most likely to be successful when the partners focus on, and actively analyse each others reasoning". (Siegler, 1998, p. 281)

The teacher's role is not diminished in this model but rather the role becomes more defined as the teacher assists the student to construct knowledge and to learn (Novak, 1992).

It was Vygotsky (1978) a contemporary of Piaget, who laid a great emphasis in the place of collaboration with others to construct knowledge. He saw that both socio-cultural dimensions and language were important to learning in formal schooling. The theory of social constructivism states the importance of a supportive learning environment. In his theory of social constructivism, Vygotsky differed from Piaget's notion of maturation and defined instead a "zone of proximal development" (1978, pp. 86-87). Within this zone, a learner was ready to construct new knowledge with the help of others; but in order to be in the zone or ready to learn the learner needed to have a foundation on which to build. The zone would change as the learner grew in knowledge and understanding (Vygotsky, 1978). One interesting aspect of this revision of the idea of *readiness* is that it allows for a much more individualised approach to curriculum planning. However individualising learner opportunities is not to say the learner is entirely alone, and Vygotsky stressed the importance of a holistic approach and the development of cultural tools, such as the technical vocabulary of the discipline. Learning does not happen in a vacuum.

Cobb (1996) is supportive of Vygotsky's emphasis on social interaction, stating that construction of knowledge is a collaborative activity and that it takes place within a particular cultural context. Knowledge making requires both culturally situated construction in a collaborative forum, and the opportunity for individuals to make meaning for themselves. Recent literature supports Vygotsky's theory of how students learn. It is in finding the balance between the individual's zone of proximal development and the mediation of learning in social interaction and cultural context that the challenges are to be found both for the learner and for the teacher. Cobb (1994) also argues that "mathematics learning should be viewed as both a process of active individual construction and a process of enculturation into the mathematical practices of the wider society" (p. 13). Students must then be encouraged to form new understandings of mathematics using interpretations of existing knowledge as an individual's frame of reference (Neyland, 1995). Mathematics is founded on the recognition that an idea can be proved to be true or false using the tools of logic. These proofs are the result of socially agreed rules and mathematical objects, such as definitions, axioms and theorems (Telese, 1999). Mathematical knowledge arises from community effort and the members of the community of mathematicians decide on the worth of any new proofs, conjectures or theories by following established protocols (Ernest, 1998). The agreements regarding these rules of mathematical proof and establishing theories arise from a shared language with common understanding of the language and its meanings (Telese, 1999). Social

constructivism and socio-cultural theory, moreover, claim that learning is enriched when students view themselves as a part of a community. When considering mathematics and social constructivist philosophy in the post modern age, Ernest (2004) argues that:

Mathematical knowledge, in social constructivist thinking, is not fixed, determined and ready made. Rather, it comprises texts made and received to persons within social/institutional settings—persons with their own histories, expectations and interpretations. Mathematics is, at heart, conversational and hence indissolubly linked to its context. And because of that, it is always shaped and delimited by the contexts of making or utterance and those of reception or interpretation. (p. 31)

## **2.5 Social Construction and Formalism in Mathematics Teaching**

Jaworski (1994) describes the mathematics teacher's dilemma as that of the Didactic and Constructivist tension. The place of formalist or direct instruction, as opposed to social constructivist approaches to teaching mathematics, has been a subject of some controversy for several decades (Ewing, 2011). The first published results from the Organization for Economic Co-operation and Development's research on creating effective teaching and learning environments at the lower secondary school level found in the 23 participating countries that there were two predominant beliefs of teaching and learning (OECD, 2009).

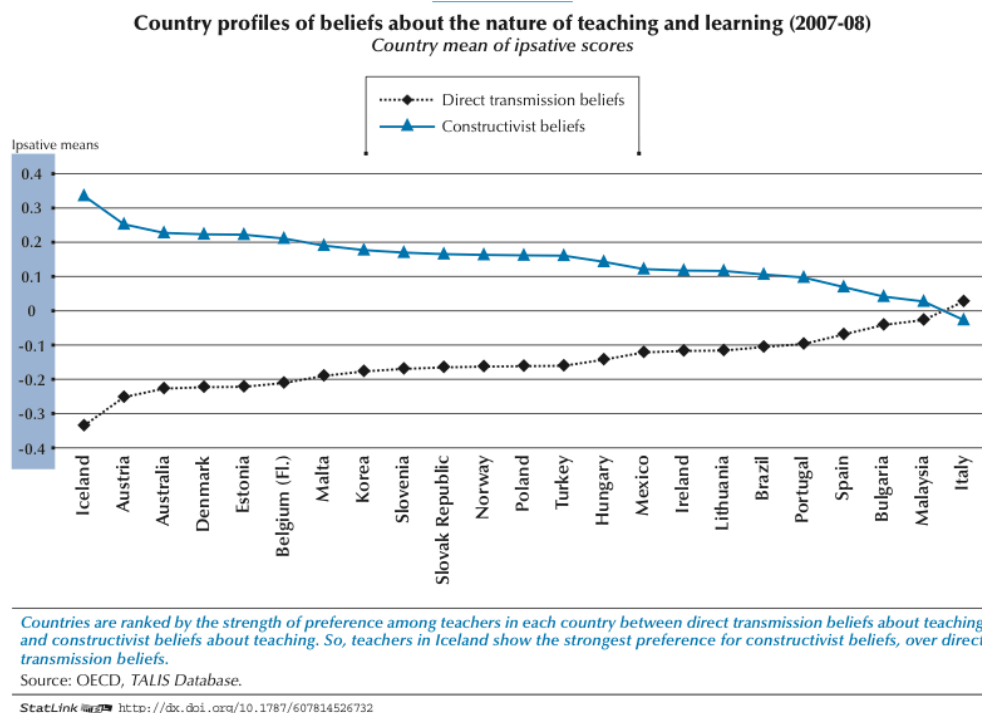


Figure 1 TALIS Graph of teachers' beliefs. (Teaching and Learning International Survey, 2009).

The direct transmission paradigm sees that the teacher's role is in transmitting knowledge and providing correct solutions. By contrast, the social constructivist framework sees that the teacher's role is that of a facilitator of active learning by students who seek out solutions for themselves. In Figure 1 the graph shows that Australian teachers have a strong preference for constructivist beliefs over direct transmission beliefs. Only Austria and Iceland of the twenty-three countries surveyed indicated a stronger preference for constructivist beliefs over direct instruction than Australia.

Research undertaken by Radford (2001), supports the notion that learning to think algebraically requires social immersion in community situations into the language, symbols and actions associated with algebra. They say "In our conception, algebraic languages (as natural languages do) might allow the students to interact between themselves and, in doing so, to elaborate new mathematical meanings and understandings" (p. 262).

Social constructivist learning theorists believe that first and foremost learning is a social activity and that the learner accommodates new knowledge through discourse and personal reflection. Underpinning this idea is the view that learning becomes the responsibility of the learner and that new knowledge must build on existing knowledge. The student

interacts with others in order to explore and understand a concept. Understanding occurs when the knowledge makes sense to the individual and the discourse of the classroom activities can help students to achieve understanding. The work of Northfield, Gunstone, and Erickson (1996) also supports collaboration as the basis of teaching and of learning. These authors note the central role that the teacher plays in student learning. Cobb (1996) identifies a trend towards socially and culturally based activity in the mathematics classroom. He and his colleagues suggest that the teacher should guide conversations with students in the classroom to engender thoughtful deliberations (Cobb & McClain, 1999). The National Council of Teachers of Mathematics report (1991) asserts that “teachers, through the ways they orchestrate discourse, convey messages about whose knowledge and ways of thinking and knowing are valued, who is considered able to contribute, and who has status in the group” (p. 20).

Sfard (2000) posits that it is the teacher as the “carrier of the tradition” who is responsible for initiating students into the language of mathematicians (p. 167). Thus the language the teacher uses in the classroom teaching early algebra initiates the student into the accepted language of the discipline. The Researcher supports Sfard’s proposal that students should not be expected to discover the meta rules of algebra. These rules and the appropriate language need to be taught and modelled by the teacher. To give too much weight to profound constructivist principles can rob the students of the powerful role the teacher can play in the students’ learning. Formal Instruction or Direct Instruction does have a role to play in teaching early algebra. Grossen and Ewing (1994) found when comparing the effect of direct instruction or constructivist approaches on teaching applications of fractions, decimals and percentages that the direct instruction group of students scored significantly higher than the students who had been working in a constructivist group. Behaviourists argue that skills should be learned through drill and practice. Brophy and Good (1986) conclude that direct instruction is the most effective instructional model for promoting achievement of the basic skills in mathematics by students in primary school. Ma (1999) argues that “to promote mathematical understanding, it is necessary that teachers help to make connections between manipulatives and mathematical ideas explicit” (p. 6).

## 2.6 Cognitive Processes and the Difficulties in Learning Algebra

I thought that mathematics ruled out all hypocrisy, and, in my useful ingenuousness, I believed that this must be true also of all sciences which, I was told, used it ...Imagine how I felt when I realized that no one could explain to me why minus times minus yields plus...

French 19<sup>th</sup> Century writer Stendhal

Understanding integers and the ability to use them fluently in calculations for learning algebra is important. Students' understanding of the structure of number and number operations is important in the primary school years if they are to make the transition to thinking and working with variables in algebra (Warren, 2003). Researchers have looked at specific types of numbers and how knowledge of these and fluency in using them prepares students for algebra. Wu (2001) argues that to develop algebraic skills and to understand algebraic concepts students must first understand fractions. It is proposed that an emphasis in the early years of schooling on developing a "deep conceptual understanding of operations and relationships with number" (Blanton, et al, 2007, p. 8) can assist students to deal with the complexities of algebra. Young students can be introduced to algebraic thinking and generalization long before they are introduced to formal algebra.

A fundamental early algebra conjecture is that when children have these experiences in elementary grades over sustained periods, they develop a much deeper mathematical foundation than children whose experiences are focused primarily on calculation procedures. As a result, early algebra develops students who are better prepared for formal algebra courses in secondary grades. (Blanton et al., 2006, p. 8)

Investigation of the difficulties encountered during the transition from arithmetic to algebra has generated numerous research projects (Filloy & Rojano, 1989; Herscovics & Linchevski, 1994; Kieran 2001). There are some interesting notational meaning contradictions that occur as the student moves from arithmetic to algebra. An example is the notation for the mixed number  $5\frac{1}{2}$ . This is a mixed numeral or number, understood to mean 5 *plus*  $\frac{1}{2}$ . In algebra, however, what appears to be very similar notation such as  $5x$  actually means 5 *times*  $x$ . (Stacey & Macgregor, 1997, p. 110).

Brain research has shown that our number system, which has been referred to as a different 'language', is dealt with in a different region of our brain from normal language. Devlin (2000) suggests that this not surprising if we accept that number symbols have their origin in counting our fingers and number words originate from ordinary language. Sousa (2008) proposes that this means that our brains understand numbers as *quantities* and not as *words*. As a result our brains automatically and unconsciously convert almost instantly the number word to a (numerical) quantity. This conversion would encompass an automatic orientation of numbers in space, large numbers to the right and small numbers to the left. It also follows that our number sense is a reflex action. We immediately attribute meaning to numbers.

However, there is no such automaticity of response when we see a letter variable, and we cognitively 'fight' the concept of a letter as a quantity when are first introduced to the idea. Instinctively we associate letters with words and objects, and not with quantities (Kieran, 1981; MacGregor & Stacey, 1993; Perso, 1993). The implication of this for the teaching and learning of algebra is that we need to address more fully the very beginnings of symbolic representation before more challenging algebraic manipulations are introduced. Extending the number system to include letters to represent a varying value is difficult as it is not automatic. Students will benefit from being taught algebraic techniques and strategies, if they are first and foremost comfortable with the notion of a variable. Hallagan (2006) found that difficulties that arise for students in making the cognitive leap from arithmetic to algebra could be alleviated by appropriate instructional strategies. This study examined teachers' developing ideas and interpretations of students' responses from selected algebraic tasks involving the distributive law and equivalent algebraic expressions.

Mc Neil, Grandau, Knuth, Stephens, Hattikudur and Krill (2006) analysed the presentation of the equal sign in different textbooks. The researchers found that children's understanding of the equal sign is "inextricably linked to the context" (p. 383). This finding is important because middle school is traditionally the point at which the transition from arithmetic to algebra is taught, and where relational understanding of the equal sign is necessary for success (Sousa, 2008). The finding also strongly suggests that teachers should present students with statements of equality in different ways to further develop their ideas of equivalence. Students in Kieran's 1988 study were more likely to learn how to isolate unknown terms by maintaining a balanced equation, while some of the students who

preferred to use the working backwards methods, like unwinding, experienced some difficulties with more challenging problems. Tabachneck, Koedinger and Nathan (1995), found that whilst informal strategies could assist students in the transition from arithmetic to algebra there were limitations when the unknown could have more than one value. The over use of 'guess and test' strategies can be inefficient, use too much valuable thinking time and cognitive resources (Tabachneck, Koedinger & Nathan, 1995). To guess and test means that the student will give an intuitive answer to the problem in the first instance. After substitution if the solution is not correct they will revise their initial answer in the light of their first result, and this process may take some time with students proposing several guesses before a correct solution is found. Informal strategies also show their limitations as the problems encountered become more complex. While these strategies are useful when used judiciously, care must be taken to balance informality with procedure. Research has shown the limitations of these informal methods and there is evidence to support the proposal that they do not support the acquisition of formal algebraic solution strategies in the classroom. There is a tension between strategy and understanding. Sousa (2008) also says that "if a learner practises a mathematical process incorrectly, but well, unlearning that process correctly is very difficult" (p. 63). To successfully find a solution, using a strategy needs to be underpinned by understanding, leading the student to generalise using algebra. Verzoni and Koedinger (1997) found that middle school students performed best on easy (one operator) problems when they were presented in a grounded story-problem format, rather than in an abstract number sentence, because the story problem brought to the fore more successful informal strategies which were founded on understanding the problem.

Sousa (2008) reports on the importance of relevance in today's classrooms, noting that:

Teachers spend about 90 percent of their planning time devising lessons so that students will make sense of the learning objective. But teachers need to spend more time helping students establish meaning, keeping in mind that what is meaningful for students 10 years ago may not necessarily meaningful for students today. (p. 56)

How does the student know if their solution to the problem is correct, and more importantly how do they find out if the reasoning behind the solution is logical? The teacher



can use class groupings and classroom discussions to provide students with opportunities to discuss their reasoning and defend their solution (Vygotsky, 1986).

As discussed in Section 2.2, students need to gain both procedural knowledge and broadly connected conceptual knowledge to understand mathematics (Hiebert & Carpenter, 1992). The Researcher would argue that this is best achieved with some attention to ‘big picture’ thinking. Knowledge of rules and procedures provides students with tools for efficient problem solving. However in learning the procedures of algebraic manipulation, students often develop only what Skemp (1978) calls ‘instrumental’ understanding of algebra. Irwin and Britt (2007) propose that where the focus for arithmetic in the classroom curriculum is on developing an awareness of the underlying structure of the operations, rather than on merely getting answers through the procedures and algorithms, it is likely there will be important positive consequences for the development of algebraic thinking. In working algebraically the student needs to see the relationship between quantities rather than just a correct answer to a calculation.

Teaching toward an overarching ‘big idea’ of number systems and number theory helps students build relational understanding by connecting what they are learning— real numbers—with what they already know—integers and rational numbers. (Woodbury, 2000, p. 227)

Students need to acquire a sense of the big ideas of algebra (Kaput, 2000), if they are to understand the why and how of this strand of mathematics. Algebraic thinking has applications for many real situations and researchers have found that algebra students generally lack a sense of the big structural ideas and therefore come to believe algebra at its core is memorizing rules and procedures (Kieran 1992). Ormond (2013), in her work on early algebra content in the Australian Curriculum, found that across the foundation years to Year Eight overarching ideas occurred at four stages. These overarching concepts are some of the ‘big ideas’ that students need to acquire in during these years.

Table 1 Australian curriculum for mathematics: The big Ideas in foundation to Year 8

Years 7/8	Generalizing number relationship with algebra
Year 6	Generalizing variation between two numbers
Years 3-5	Generalizing pattern in number
Years, Foundation, 1 & 2	Familiarity with pattern in number

*Note.* Adapted from Ormond (2013)

Two studies, one using animations of situations (Nathan et al., 1992) and one using concrete arithmetic instances (Koedinger & Anderson, 1997), have shown that instruction that bridges formal algebra instruction to previously grounded representation does help students learn processes such as algebraic modeling of verbally presented relations. These two studies are constructivist in their approach as students transition from the concrete to the abstract; constructing meaning for themselves out of the experiences. The two studies, although they differed in the type of grounded representations used, yielded similar results, suggesting that a crucial feature of success was the role of grounded intermediate representations in students' learning.

The connections made by the student in developing their own thinking mathematically can be enhanced by appropriate teacher practice. Care must be taken to ensure that students form the relationship between the ideas. This is also supported by the work of Resek and Kirshner (2000). Both structure and process are important. Sfard (1991) suggests that the attention given to the structured approach over the process skills is due to an over-reaction to behaviourism. In his position as chair of the Adding it Up mathematics learning study committee, Kilpatrick (2001) states that:

If students are to become proficient in mathematics, teaching must create learning opportunities both constrained and open. Mathematics teaching is a difficult task under any circumstances. It is made even more complicated and challenging when teachers are paying attention simultaneously, as they should,

to the manifold paths mathematics learning can take and to the multifaceted nature of mathematics itself. (2001, p. xiv)

It is the teacher's role to structure the learning environment and the learning opportunity for the student if the student is to bridge the 'cognitive gap' (Herscovics & Linchevski, 1994). Researchers have shown that students who do not know and understand the basic rules which govern the four operations of number have great difficulty in coping with the abstractions of algebra. The teaching needs to be carefully planned to support the learner to make meaning and to practise and develop skills. Research into what appears to be effective practice is a valuable resource in teacher development, and this will be explored in this thesis.

## 2.7 Teachers' Beliefs

Teachers' beliefs are a key factor in determining the success of curriculum reform (Battista, 1994; Koehler & Grouws, 1992). According to a constructivist view, any attempts made to develop teaching practices must attend to teachers' beliefs (Pajares, 1992). Swan (2005) cites Ernest (1991a, b) and Askew et al. (1997) thus:

Ernest suggests that a teacher's belief system had three components; the teacher's conception of the nature of *mathematics as a subject for study*, the teacher's view of *the nature of mathematics teaching* and *the process of learning mathematics*. Askew et al. (1997) characterized the orientation of teachers of teachers towards each of these components as *transmission* (T), *discovery* (D), or *connectionist* (C). (Swan, 2005, p. 59)

Teaching from a transmission orientation is defined as traditional practice. Mathematics is seen as a series of rules and truths which the teacher transmits to the students, and students are expected to practise individually until fluency is achieved. It is teacher centred practice. Teaching from a discovery orientation is defined as student centred practice. Mathematics is seen as a human creation. The teacher adopts the role of facilitator. Students make meaning for themselves through individual exploration and reflection. They learn often by trial and error to build on their existing knowledge. Teaching from a connectionist orientation is defined as a blend of teacher and student centred practice.

Mathematics is seen as a network of ideas. Through collaborative discussion the teacher aids the student to construct his or her knowledge and understanding of mathematical concepts (Askew, et al., 1997).

The research undertaken by Swan (2005) was part of a national project aimed at promoting better understanding of algebra by students studying for the General Certificate of Secondary Education Examinations in England and Wales. Questionnaire II was used to relate the teachers' reported practices to their predominant beliefs. Swan found that there was inconsistency between what teachers believe and what they practise in teaching algebra. Swan (2005) cites Wilson and Cooney (2002), who found that in attempting to describe beliefs about teaching and learning one must recognize the situated nature of the data:

Attention must be paid to constraints of time and curriculum design and other difficulties under which teachers work. However, teachers' personal beliefs and theories about mathematics and the teaching and learning of mathematics are widely considered to play a central role in their teaching practices. (Swan, 2005 p. 59)

However, Kyeleve and Williams (2006) found that if teachers believed in the efficacy of change to curriculum or pedagogy they then welcomed it regardless of external imperatives. To contemplate teachers' beliefs and practices can be challenging (Handal, 2003; Koehler & Grouws, 1992), and if any change in practices is to occur then teachers' beliefs must be explored, and a platform found from which future professional development of teachers may be advanced. Swan used the questionnaires primarily to select a working sample from a very large group of teachers. The smaller, working group comprised teachers who had widely differing beliefs about what mathematics is, what learning mathematics is and what teaching mathematics is. The questionnaires were used before any units of professional development.

The exact nature of the relationship between beliefs and practice is as yet unclear and raises the question of the effect of beliefs on practice and practice on beliefs. Buzeika (1996, p. 96). Can change of practice change beliefs? Why do teachers do what they do? If we have a model of practice then we can find out how their beliefs fit in. Romberg (1994) suggests that mathematics teachers often view their job as showing a few standard facts and algorithms to students and later, "supervising some drill and practice" (p. 314).

A limited view of what mathematics teaching is can also be found outside the mathematics community. Boero and Szendrei (1998) claim:

It is necessary to overcome the false idea, so frequently shared by those not in the mathematics community, that effective teaching of mathematics is essentially based on good “technical knowledge” of the topics to be taught and the quality of the teacher as a self-made artist. (p. 209)

Nathan and Koedinger (2000a) support the notion that teachers’ beliefs about student ability and learning greatly influence their instructional practices, so they set out to observe the effect of problem features on high school teachers’ predictions about students’ difficulties in solving algebraic problems. They found that the teachers in the study incorrectly predicted the difficulty students experienced, believing that symbolic equations would be easier than worded and story problems. To understand teachers’ beliefs about prospective problem difficulty is of value because these beliefs are likely to affect teachers’ instructional planning and the design of their assessments (Borko & Shavelson, 1990; Carpenter et al., 1980). Hargreaves (1994) speaks of the need for an honest and thorough examination of what teachers do and why the teacher’s voice needs to be heard.

In the 2009 Teaching and Learning International Survey (TALIS) there was a relationship found in TALIS countries between teachers’ beliefs and their classroom practices. In particular, teachers who employ student-oriented practices are more likely to be those who take a ‘constructivist’ view of teaching. On the other hand, there was no consistent pattern to the association between teachers’ beliefs and more structured lessons and teaching. It was also found that both ‘constructivist’ and ‘direct transmission’ beliefs were positively associated with teacher self-efficacy in most TALIS countries. Even though these are different views of teaching, this result indicates that holding any strong view about technique tends to be associated with confidence in one’s own effectiveness. Warren, Cooper and Lamb (2006) found in their work in the Early Algebra Project, which interrogated teachers’ conceptions of a zone of proximal development, that the teachers’ beliefs about their teaching practices could be distorted by poor pedagogical content knowledge.

## 2.8 Mathematics Knowledge for Teaching

In Shulman's (1986) seminal work, *Those Who Understand: Knowledge Growth in Teaching*, he suggested that there are three forms of knowledge for teaching and seven categories of professional knowledge that teachers require in order to teach. Shulman was dismayed by the poor opinion of teachers and teaching voiced by some and his subsequent identification of forms of knowledge and categories of knowledge demonstrated the complexity of the role of the teacher and the knowledge required to teach. The forms of knowledge he identified are the principles, maxims and norms that influence the teacher in the practice of teaching. The categories of professional knowledge include:

1. General Pedagogical Knowledge;
2. Knowledge of Content;
3. Curriculum Knowledge;
4. Knowledge of Learners and their Characteristics;
5. Knowledge of Educational Contexts;
6. Knowledge of Educational Aims, Goals, Purposes; and
7. Pedagogical Content Knowledge (PCK). (p. 14)

Prior to Shulman's work the majority of research in teaching and learning had focussed on process and outcomes. What did the teacher do and what were the results for the students? In this work, Shulman turned the focus to the teacher's knowledge for teaching. Knowledge of a discipline alone is not enough when teaching for student understanding, and therefore Shulman created the term PCK (Pedagogical Content Knowledge) which is the specific content knowledge as applied to teaching. The relationship between the teacher's subject matter knowledge of mathematics and student learning has been well documented by Researchers (Askew, Brown, Rhodes, Johnston & William, 1997; Charalambous, Hill & Mitchell, 2012; Hattie, 2012; Hill, Ball & Shilling, 2008). Teachers need to know their subject in some depth and be mindful of detail (Ball, 1991; Ma, 1999). Ball, Hill and Bass (2005) found that teachers' fluency in the use of mathematical symbols and language does assist students' learning.

The conceptual demands of teaching mathematics are different from those of other subjects and therefore researchers have sought to develop a revised framework, which could

be used to analyse pedagogical content knowledge in relation to mathematics teachers. Taking three of the categories of content knowledge for teachers devised by Shulman; Ball, Hill and Shilling (2008) created a model which attempted to further refine the concept of mathematical knowledge for teaching. The impetus for this was to study “knowledge in and for teaching rather than ... the teachers themselves ” (p. 394). In this model (see Figure 2) Pedagogical Content Knowledge (PCK) and Subject Matter Knowledge (SMK) are two domains. Curriculum Knowledge is subsumed in Pedagogical Content Knowledge (PCK) alongside Knowledge of Content and Students (KCS) and Knowledge of Content and Teaching (KCT). Within the domain of Subject Matter Knowledge (SMK), Common Content Knowledge (CCK), Specialised Content Knowledge (SCK) and Knowledge at the Mathematical Horizon are to be found. In this model the authors accept that the boundaries between each category of knowledge are not exact and that these lines are necessarily blurred.

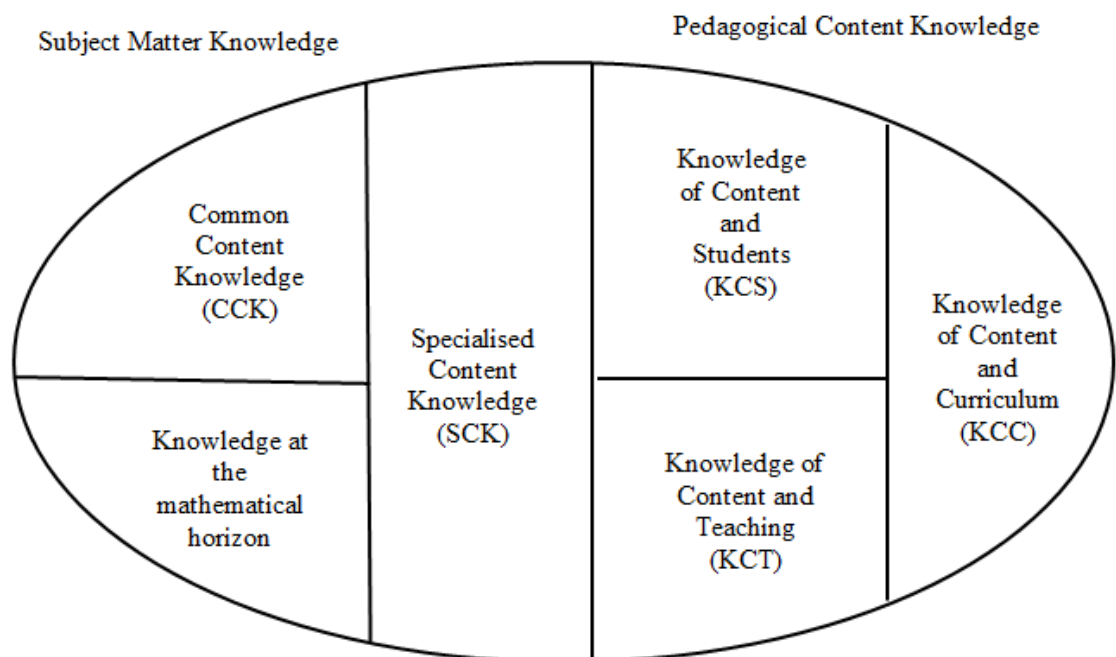


Figure 2 Mathematical knowledge for teaching (Hill, Rowan, Ball, & Schilling (2008).

Silverman and Thompson (2008) propose that to bring continual improvement in teaching the emphasis in teacher professional development should be in teachers’ SMK rather than focussing solely on PCK as discussed earlier. In focussing on understanding the teacher needs to know the big ideas behind the content in the mathematics they are teaching and know how each unit of content contributes to the whole. Silverman and Thompson (2008) argue that it is not until the teacher transforms their knowledge of mathematics alongside

their pedagogical content knowledge that the teacher has developed their mathematical knowledge for teaching (MKT).

The Knowledge Quartet (KQ) was developed first as a theory, and then as a framework by Rowland, Turner, Thwaites and Huckstep (2009) to assist the development of the knowledge of mathematics for teaching in beginning teachers. They argue that it is in classroom observations that mathematical content knowledge can best be identified and that teacher development is enhanced by teacher reflection. The Knowledge Quartet is supported by socio-cultural theory. Ball, Hill and Shilling's (2008) model of mathematics knowledge for teaching attempts to classify different kinds of mathematical knowledge within the domain of Subject Matter Knowledge. Common Content Knowledge (CCK) is the knowledge that any citizen may be expected to know. Specialist Content Knowledge (SCK) is the knowledge of mathematics and skills a teacher needs for the classroom. Knowledge at the Mathematical Horizon (KMh) is knowledge of mathematics currently being developed.

Turner suggests that KQ and MKT (Ball et al., 2008a) offer complementary frameworks from which to analyse teacher knowledge and practice. Table 2 outlines the relationship between the Knowledge Quartet framework, Shulman's forms of knowledge and categories of mathematical knowledge, and Ball, Thames and Phelps' more recent work defining categories of knowledge.



Table 2

The relationship between knowledge quartet framework and (a) Shulman's forms of knowledge; (b) the categories of mathematical knowledge of Shulman, and of Ball et al.

Knowledge Quartet Framework	Shulman (1986) Forms of Knowledge	Shulman (1987) Categories of Knowledge	Ball, Thames and Phelps (2008) Categories of Knowledge
<b>Foundation</b> adheres to textbook; awareness of purpose; concentration on procedures; identifying errors; overt subject knowledge; theoretical underpinning; use of terminology.	Propositional Case Study	Subject Matter Knowledge (SMK) Theoretical Pedagogical Knowledge (PCK) Curricular Knowledge (CK)	Common Content Knowledge (CCK) Specialised Content Knowledge (SCK)
<b>Transformation</b> choice of examples; choice of representation; demonstration.	Propositional Case Study	'Active' Pedagogical Content Knowledge (PCK)	Knowledge of content and teaching (KCT) Knowledge of content and learners (KCL)
<b>Connection</b> anticipation of complexity; decisions about sequencing; recognition of conceptual appropriateness; making connections between concepts; making connections between procedures.	Propositional Case Study	Subject Matter Knowledge (SMK) Curricular Knowledge (CK)	Common Content Knowledge (CCK) Specialized Content Knowledge (SCK) Knowledge of Content and Learners (KCL)
<b>Contingency</b> deviation from agenda; responding to children's ideas; use of opportunities; teacher realisation.	Strategic Knowledge (which can only come into play in the act of teaching which involves making appropriate strategic decisions by drawing on relevant <i>propositional</i> and <i>case study knowledge</i> )	Subject Matter Knowledge (SMK) 'Theoretical' and 'active' Pedagogical Content Knowledge (PCK)	Common Content Knowledge (CCK) Specialised Content Knowledge (SCK) Knowledge of Content and Teaching (KCT) Knowledge of Content and Learners (KCL)

*Note:* Adapted from Turner, F. (2012).

In the Knowledge Quartet (KQ) model created by Rowland et al., (2009), the classification is of mathematical knowledge that surfaces in teaching. In the knowledge framework the teacher's knowledge for teaching is divided into four classes of knowledge:

Foundation, Transformation, Connection and Contingency. Each describes the knowledge needed for teaching mathematics.

The Researcher would propose that both the Knowledge Quartet model and the Knowledge Framework model have useful insights to offer and that each can enrich the other when examining the complexity of the knowledge required for teaching mathematics. The KQ is a framework from which the teacher can reflect on their actual practice in the light of Shulman's forms of knowledge. The categories of knowledge identified by Shulman (1986) and then by Ball, Thames and Phelps (2008) can assist the teacher in addressing the specific area of knowledge they may wish to improve. This narrowing of focus facilitates self-directed professional development. Darling-Hammond and McLaughlin (2011) have suggested that teachers learn in the same manner as students and that professional learning for teachers should therefore be teacher-centred.

Zhang and Stephens (2013, p. 489) developed a construct to discern different levels of teacher capacity in an exploratory study involving 120 teachers from both China and Australia, as teachers "help students connect arithmetic learning and algebraic thinking" during a time of curriculum reform. The term teacher capacity they assert has arisen from recent literature on curriculum reform. In their model they identified four criteria on which to assess teacher capacity.

The components of their model were defined as

- (i) knowledge of mathematics
- (ii) interpretations of intentions of official curriculum documents
- (iii) understanding of students' thinking
- (iv) design of teaching to foster the underlying mathematical ideas. (p. 483)

The researchers found that the design of teaching, informed by the three other criteria emerged as the critical dimension for the implementation of curriculum reform (Zhang & Stephens, 2013).

The Researcher made reference earlier in this review to the importance of language in learning and teaching early algebraic ideas. An Italian project called Arithmetic Pathways Towards Favours Pre-Algebraic Thinking (Malara & Vavarra, 2003) investigated the multifaceted relationship between theory and practice. This study categorized teachers' decision making as teachers' practice. The teachers' use of language was a focus and video was used to record lessons. Teachers' were required to transcribe their own lessons. The transcripts were then analysed jointly with the researchers. In the final phase of the project a, "global revision of the teachers' transcripts is made and this turns out to be the climax of the whole experience"(Malara & Vavarra, 2003, p. 550). The resultant units of work compiled by the teachers in the study proved to be useful models of teaching for others to use. The researchers propose that involvement in the project raised teachers' awareness of their actions and the repercussions of those actions as teachers in the classroom.

A study which investigated Colombian mathematics teachers' conceptions of beginning algebra and their conceptions of their own teaching practices found that the teacher's own way of " knowing beginning algebra represented the basis for the teacher's pedagogical purpose behind her/his preferred teaching practice" (Agudelo-Valderrama, Clarke & Bishop, 2007, p. 87). The 13 teachers in the study were Grade Eight teachers because this is the year when algebra work began in the system. The results from the investigation found that the teachers' did not see the correlation between their own understanding and their pedagogy.

In each of the studies included in this section of the literature review the teacher and the teachers' practice have been at the centre of the research. The most recent work by Zhang and Stephens (2013) found that the key aspect in teaching for students' transition from arithmetic thinking to algebraic thinking is in the teachers planning.

## **2.9 Conceptual Framework**

An elaborated conceptual framework derived from the literature is shown in Figure 3. The conceptual framework is based on the premise that the pedagogical approach adopted by teachers is influenced by their beliefs and their theories of knowledge and learning. The purpose of the study is to investigate teachers preferred balance of didactic and constructivist strategies for teaching beginning algebra.

Through reviewing the literature it can be observed that algebra is seen as an important area of study and that researchers have ensured that it has been rigorously investigated in recent years. Current findings indicate that work done by teachers in the early years of schooling has a significant impact on students' engagement and levels of success in early algebra. In the literature review it is recognized that algebra is a challenging strand of mathematics to study and the difficulties students can experience were considered.

Nonetheless, the review of current literature on mathematics teaching practice revealed that there is little in the literature that shows how teachers' beliefs influence the practice of teaching early algebra in the transitional years of the Year Seven and Year Eight classrooms. This study contributes to addressing this gap in the literature.

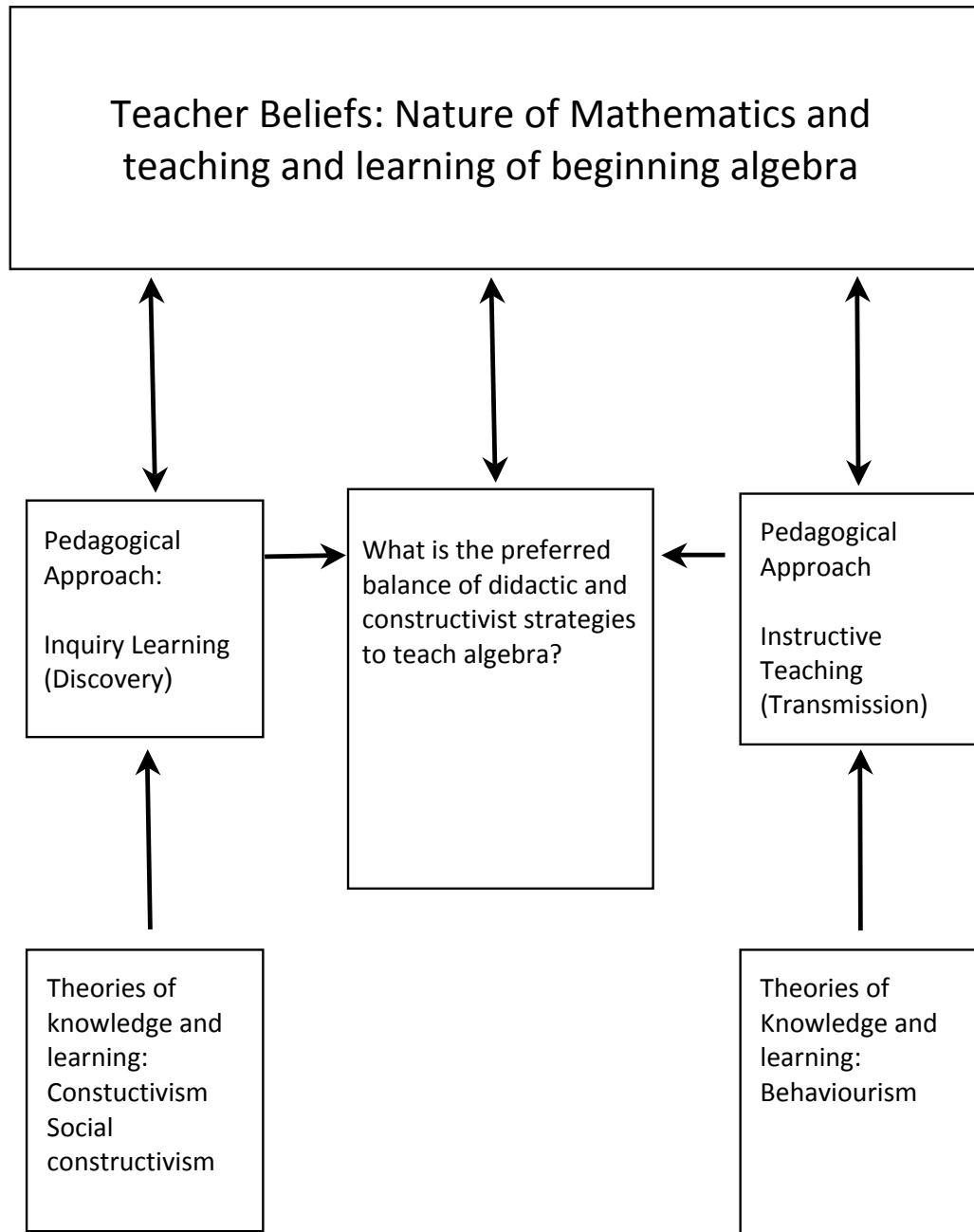


Figure 3 Conceptual framework

# Research Approach

## 3.1 Introduction

In this chapter the reasons for the methodological approach taken in the study will be outlined and supported by literature. Each of the methods used in this mixed methods study are explained and justified. The design of the research, including the data-gathering instruments employed to gather both quantitative and qualitative data, are described. Finally the validity and reliability of all data gathered is discussed.

## 3.2 Methodological Issues

The background to the methodology used in this study is described in this section. The organising principles are discussed and the reasons for adopting a mixed methods approach to gather data are outlined. The theoretical framework for the study is included.

### 3.2.1 Theoretical approaches to research

Theoretical perspectives in any research project are underpinned by the Researcher's beliefs in a particular epistemology. The literature presents different worldviews or different paradigms about the nature of knowledge and obtaining knowledge. Kuhn (1970) discusses the concept of a paradigm and a paradigm shift. He envisages that changes to beliefs and practice in research would be the result of researchers' engagement in the field. Since Kuhn's early work, there has been an explosion of literature examining the theoretical foundations of recent paradigm shifts in research, particularly in the social sciences (Babbie, 2004; Black, 1999; Miles & Huberman, 1994; Neuman, 2006; Thomas, 2003).

Guba and Lincoln (2005) identify five of these paradigms based on different assumptions of the nature of knowledge. To answer the research questions in this study the Researcher has used a constructivist paradigm. The reason for this is determined by the nature of the research. Firstly there is an exploration of the teachers' beliefs about learning and

teaching early algebra. The Researcher examines what exists. Further analysis by the Researcher explores the teachers' practice with reference to their espoused beliefs. Synthesis of the data results in the construction of new knowledge.

As suggested by Greene (2006), there is value in comparing and contrasting the inferences that emerge from examining the findings of a study from multiple worldviews and perspectives.

### **3.2.2 Rationale for methods**

A mixed methods approach was taken in the collection and analysis of the data gathered from the research. The choice of this approach aligns with the complex nature of teaching and learning in the classroom (Kumar, 2011). Thomas (2003) argues that the use of mixed methods adds strength to findings by allowing for different perspectives to be examined. The advantages of a mixed method approach can be summarized as follows.

- Data can be checked from different sources.
  - The use of both quantitative and qualitative data provides a comprehensive picture of the phenomenon being explored.
  - Quantitative data can support the findings of qualitative data and likewise qualitative data can be used to support the findings of quantitative data.
  - Qualitative data can assist the interpretation of relationships between variables.
- (Punch, 1998)

Table 3 describes the characteristics of both qualitative and quantitative research methods which have been used in this study.

Table 3 Characteristics that distinguish the methods of research. (Leedy & Ormrod, J.E, 2001).

Question	Quantitative	Qualitative
What is the purpose of the research?	To explain and predict To confirm and validate To test theory	To describe and explain To explore and interpret To build theory
What is the nature of the research project?	Focussed Known variables Established guidelines Static design Context-free Detached view	Holistic Unknown variables Flexible guidelines Emergent design Context-bound Personal view
What are the methods of data collection?	Representative sample, large Standardised instruments	Informative, small sample Observations, Interviews
What is the form of reasoning used in the analysis?	Deductive analysis	Inductive analysis
How are the finding communicated?	Numbers Statistics, aggregated data Formal voice, scientific style	Words Narratives, Individual quotes Personal voice, literary style

The choice of a case study design for this research is supported by the work of Yin (2009) who suggests that there are three criteria for deciding to use case study approach. Each of the three criteria is met in this study. Firstly, questions are asked about how and why teachers act the way they do when they teach algebra. How do they plan their lessons to teach students the beginning algebra concepts? Furthermore, why do teachers employ a particular set of strategies to achieve their teaching objectives? Secondly, it must be noted that the Researcher had little or no control over the events which unfolded during the practice. The individual teacher's beliefs and practices are examined, but there are variables within the classroom practice, which are beyond the Researcher's control. Thirdly, the Researcher



explored teaching and learning within an authentic context. The data gathered are from practising teachers in the teaching of early algebraic concepts in Year Seven and Year Eight classrooms.

### 3.2.3 The application of the multi method design for this research.

For the purposes of this study there were two distinct phases of data gathering. In Phase One, the four teachers in the study completed two questionnaires. The first was used to collect information about the teachers' beliefs about the nature of mathematics, teaching, and learning. The second questionnaire was used to collect information about the teachers' beliefs about their practice. In Phase Two data was gathered from semi-structured interviews, which were conducted at the beginning and at the end of the study. Classroom observations and videotaping of four lessons by each of the teachers were recorded. A focus group meeting with all four teachers was held on the 5<sup>th</sup> of November 2012. The teachers' reflections on their practice at the completion of teaching a unit of work in early algebra were used to explore their beliefs and their practices. Table 4 shows the two phases of the data-gathering period.

Table 4 Phases of data gathering

PHASE	Data Gathering Instruments	Data
1	Questionnaire I Questionnaire II	Quantitative
2	Interview 1 Lesson observations Focus Group meeting Teachers' reflections Interview 2	Qualitative

### **3.3 Research Design**

#### **3.3.1. Overall design of the study**

The study examined teacher beliefs and how these inform the approach adopted in the teaching of beginning algebra. The research questions were:

1. What were the teachers' beliefs about the teaching and learning of algebra?
  - (d) How did they view the educational importance of algebra?
  - (e) How did they consider the teaching of number?
  - (f) How did they choose to balance transmission, connectionist and discovery strategies to teach beginning algebra?
2. How does reflection upon these particular beliefs and practices inform the teachers' methods for fostering the successful conceptual development of key beginning algebraic ideas?

The relationships between these questions and the research methods are outlined in Figure 4.



### 3.3.2. Data collection methods

The following sections describe the data collection instruments employed in this study. A detailed report of the methods used in each phase of the data-gathering period and subsequent analysis are also provided. The Researcher contacted AISWA for permission for the research to proceed. The Researcher then sought advice from mathematics consultants at AISWA on teachers who may be interested in being a part of the research project. Some 20 teachers were contacted by email or by phone and given information about the research. In some instances the teachers contacted were also Heads of a Mathematics Department at the school and they communicated information about the research to their staff and asked if any of the Year Seven teachers were interested in becoming involved. This approach to find interested teachers proved difficult. Of the two teachers who said that they would participate one withdrew very early in the process. The Researcher received very positive responses in terms of support for the research from the teachers who had been contacted but there were several reasons given why it was not possible for them to be a participant at this time. Some of the reasons given for non- participation were as follows:

- (i) The school had graduate teachers teaching Year Seven
- (ii) The school had teachers who were new to teaching Year Seven
- (iii) The teacher of Year Seven was new to the country
- (iv) Lack of time available to devote to the study given the recent changes to the curriculum

The Researcher then contacted individual school principals who were sent information detailing the proposed study and a letter asking them to grant permission for teachers in their schools to be a part of the project. An information letter and letter of invitation was sent to teachers of Year Seven mathematics, outlining the purpose and process of the proposed research. Teachers were asked to nominate to be a part of the study. The four teachers who volunteered came from different schools with different levels of available resources. Three teachers of Year Seven and one teacher of Year Eight mathematics were involved in the study. The work planned for the Year Eight class was similar to that of a Year Seven class and therefore this change did not detract from the purpose of the study, namely to explore the teaching of beginning algebra.

Data collection began after ethical clearance had been granted by the ECU Ethics Committee. The data-gathering period began in Term 1 and concluded early in Term 4 of the 2012 school year. It was important to respect the teachers and the students involved and therefore all points of view were taken to have equal validity. A high level of confidentiality was maintained throughout this project and all data collected was anonymous. The teachers were allocated a pseudonym and the de-identified data were stored in a locked filing cabinet.

### **3.3.3. Quantitative data collection and analysis**

As recorded in the review of literature, Swan (1996) cites Ernest thus:

A teacher's belief system about mathematics teaching has three components: the teacher's conceptions of the nature of mathematics as a subject for study; the nature of mathematics teaching; and the process of learning mathematics.  
(p. 59)

These beliefs were explored using Questionnaire I, (Appendix 1) which explored the teacher's beliefs about the nature of mathematics and teaching and learning. In Questionnaire II (Appendix II) Swan asked teachers to describe the frequency of 25 classroom behaviours on a five point scale. Thirteen of the behaviours may be described as teacher centred and 12 as student centred. The responses gathered from the beliefs questionnaires provide a background from which the interviews and observations are explored in this study. The correlation between different belief paradigms and practice are identified in the analysis.

SPSS 11.0 was initially used to store and analyse the data. The data were coded and entered into a database, and then examined using the analysis package. However, the small number of participants in the study made this approach unproductive. The Researcher abandoned the use of the SPSS software and instead used Microsoft Excel spreadsheets and graphs to analyse the data. This analysis is described in detail in the next chapter. In the constructive paradigm that underpins this study the statistical importance of this quantitative data is to support the qualitative findings.

### **3.3.4. Qualitative data collection and analysis**

Phase Two of the study involved the collection of the qualitative data. The qualitative data derived from this research were used to compile four individual case studies.

Four methods of qualitative data collection were used in the study. Firstly, semi-structured interviews were conducted at the beginning and at the end of the data-gathering period. Each interview was recorded and the Researcher took notes. The second set of questions varied considerably from the initial proposed questions. This is recognized as being necessary as the design of a study emerges or evolves over time (Wellington, 2000). As the Researcher became more familiar with the beliefs and practices of the teachers involved in the study the need to change the questions became apparent. The second interview was also the Researcher's final opportunity to answer any outstanding queries or to seek clarification of any points to assist in answering the research questions. The length of interviews varied, and was governed by the teachers' responses to the questions. The interview questions are attached as Appendix 3.

Secondly, observations of a series of four lessons by each teacher were recorded. This was originally planned as a selection of non-sequential lessons, spread evenly throughout the data-gathering period of the research. However, this was not possible in each case. The Researcher recorded the lessons with one camera, which was set up on a tripod at the rear of the classroom and followed the teacher as he or she taught. The camera also captured notes from an interactive whiteboard, when these were used in the classroom, and notes from the whiteboard. The teachers' and students' anonymity was protected. Teachers were given pseudonyms and student names were not recorded in transcripts. The video taken was used only for this research project and adheres to ECU ethics requirements. Student work samples were collected and students' names were not used.

Use of video to collect data provided the Researcher with an accurate record of each lesson. The lessons could then be revisited many times at a later date. Transcription of segments and assigning of codes were facilitated by the use of the video records.

Thirdly, a focus group meeting was held on the 5<sup>th</sup> of November 2012. The discussions at the meeting were recorded and transcribed to provide data for reflection and contribution to the building of the case. The meeting involved:

1. **A short power point presentation.** The Researcher described the three orientations to teaching approaches as described by Askew et al. (1997). The idea of a balanced approach, in terms of strategies used in teaching was raised by the Researcher.
2. **Supporting documentation.** The framework of a unit of work on early algebra that had been compiled by the Researcher was provided and discussed. Each lesson linked to the Australian Curriculum and the each lesson was reinforced by the inclusion of research findings and relevant research commentary. The purpose of this supporting documentation was to provide the teachers with material for reflection on their practice. (See Appendix 7)
3. **Discussion of three lessons from the unit of work on early algebra.** Lesson 3 was identified by the Researcher as essentially working from a transmission approach. Lesson 16 was identified by the Researcher as based on an inquiry or discovery approach. Lesson 6 was identified by the Researcher as based on components of both transmission and discovery approaches. Identification of the major components of the three lessons was a goal of the discussion, as was the process these components were seen to contribute to the teaching objectives of the teacher within the unit of work in early algebra.
4. **List of strategies.** A list of strategies that the Researcher had observed and classified at the conclusion of the video phase of the data collection, was also provided to teachers for discussion.

Fourthly, teacher reflections were recorded on paper. The original intention had been to use an online discussion board to collect teacher reflections about their teaching of a unit of work on beginning algebra. Specifically it was hoped to find out why teachers chose to use a particular strategy. However, early in the data-gathering phase of the study the idea of using a discussion board was abandoned. The reason for this change of data-gathering approach was

that the Researcher had been advised that the use of an online discussion board might cause problems for some school networks. At the completion of the unit of work in beginning algebra the teachers were asked to write a response to four questions, (Figure 5) which asked them to reflect on their teaching practice and their students' learning.

1. In the recent unit of work on algebra, what would you say were the most successful lessons?
2. What do you think made the difference?
3. Were there any surprises, that is, questions or insights offered by students you did not expect? If so why do you think this happened?
4. Is there anything you would do differently? If you answer yes to this question can you elaborate by saying what you would change and why?

Figure 5 Questions for reflection by teachers

Yates described the stages of making meaning from qualitative data thus:

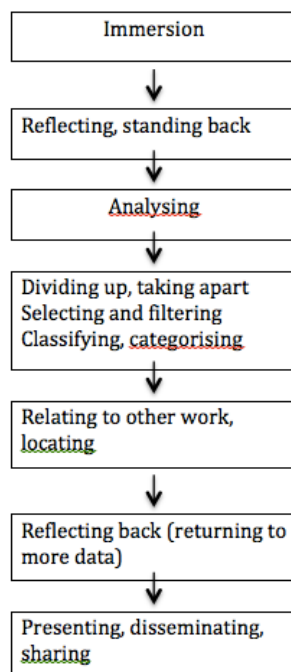


Figure 6 General stages in making sense of qualitative data. From: (Yates, 2004)

Miles and Huberman (1994) describe the analysis of data as occurring in three stages. These are data reduction, data display and conclusions and verification.



Methods of analyzing the qualitative data gathered by four different methods are summarized below.

Transcripts were coded and organized into themes. Wellington (2000) suggests that:

Some categories are pre-established while others are derived from the data, that is a mixture of a *priori* and a *posteriori*. This is probably the most common and, in my view the most rational approach, to analyzing qualitative data. In my experience it almost always happens whether people admit it or not.  
(p. 42)

During lesson observations the Researcher took notes that provided an additional source of data. Erickson (1982) suggests that video is a data source from which data must be separately constructed. The Researcher must decide what to look for within what time frames. The software program Artichoke (Fetherston, 2013) was used to assist the transcription, coding and identification of themes. Pea and Hay (2003) distinguished 10 purposes of video research that may or may not be helpful tools in the process of such analysis. These purposes are as follows; acquisition; chunking; transcription; way-finding; organization and asset management; commentary; coding and annotation; reflection; sharing and publication, and presentation.

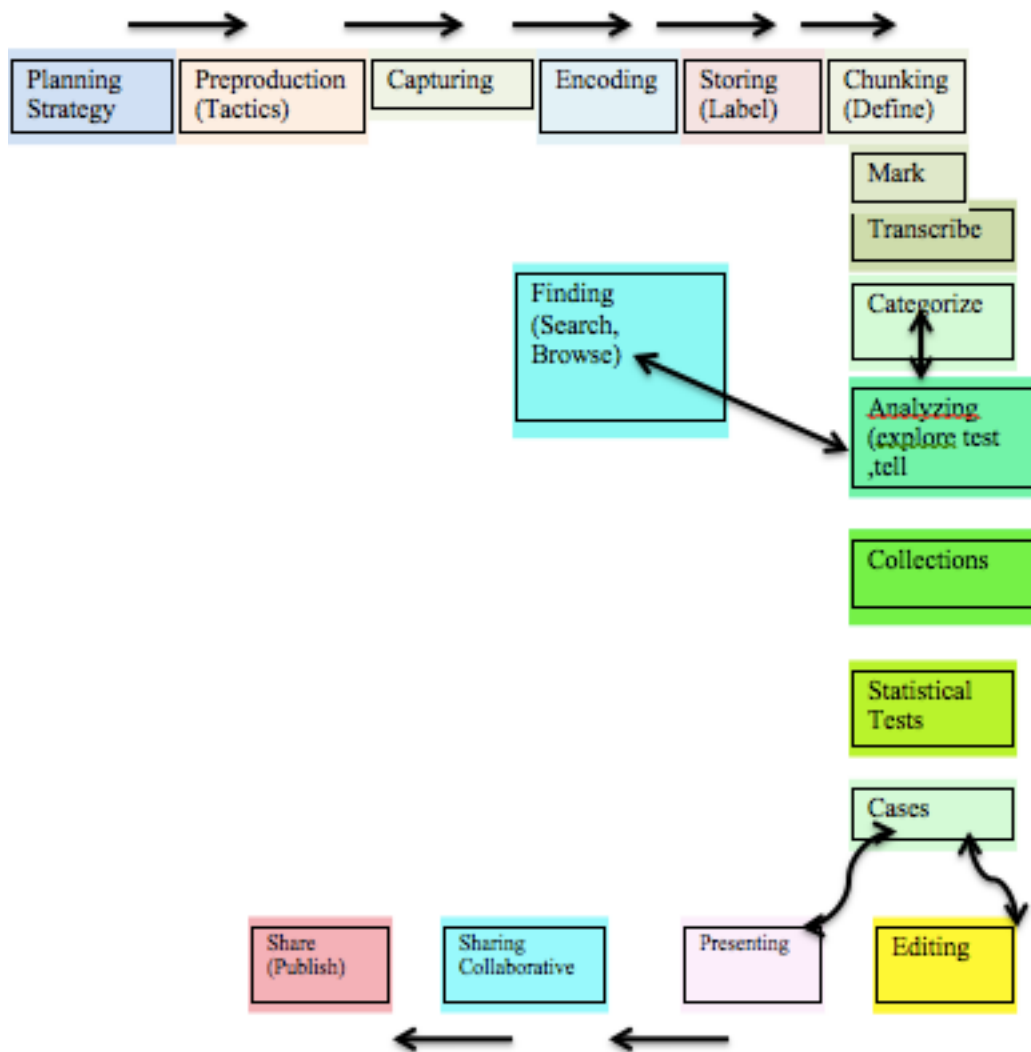


Figure 7 Diagram of video research workflow process used in this research project. (Adapted from Pea & Hoffert, 2007).

Focus group meeting transcripts were analysed using a grounded theory approach. Dick (2005) defined the progression of steps required as the developing process of this approach. The transcript was coded, categorized, sorted and constantly compared with both qualitative and quantitative data. Comparisons were made within the case of each teacher, and also across cases. Reflections by teachers were also analysed using a grounded theory approach. The responses were coded, categorized, sorted and constantly compared with both qualitative and quantitative data. Again comparisons were made within and outside the cases.

### 3.3.5. Ethics

The four teachers were given information about the data gathering procedures and the assurance that their anonymity would be preserved. The parents and guardians of the students in each class were given information letters and letters of consent to sign. The students in each of the four classes were given information letters and consent forms to sign. All those involved in the study were informed that they could withdraw at anytime. The data gathered were stored in a secure location in a locked filing cabinet. All possible identifiers were removed from student work samples. Digital video files were secured on a password protected computer.

## 3.4 Validity and Reliability of the Data

This section discusses the validity and reliability of the research for both the quantitative and qualitative facets of the study.

### 3.4.1 Validity, generalizability, and reliability

This is a descriptive, exploratory process, and the thesis rationale supports the validity of the research.

Table 5 Quantitative research criteria. (adapted from Borrego, Douglas & Amelink, 2009).

Quantitative Research Criteria	
Validity	Project and instruments measure what is intended to be measured
Generalizability	Results are applicable to other settings, achieved through representative sampling
Reliability	Findings are replicable or repeatable
Objectivity	Researcher limits bias and interaction with participants

Table 5 shows an interpretation of quantitative research criteria by several researchers. For the purposes of this research, the instruments used, Questionnaires I and II,

were grounded in the literature about algebra teaching and beliefs and practice. The instruments have been used for similar studies. A recent study sought to examine the validity and reliability of Questionnaire II. This instrument was used to measure mathematics teachers' teaching practices. Maat, Zakaria, Nordin and Meerah (2011) validated the instrument by both "exploratory and confirmatory factor analysis" (p. 2093).

The current study explored the beliefs and teaching practices of four teachers from four schools. Given the limited sample size, caution should be exercised in any generalization of findings.

### 3.4.2 Validity, reliability of the qualitative component of the research

Table 6 Qualitative research criteria. (adapted from Borrego, Douglas and Amelink, 2009).

Qualitative Research Criteria	
Credibility	Establishing that the results are creditable and believable
Transferability	Applicability of research findings to other settings achieved through thick description
Dependability	When Researchers account for the ever-changing context within which the research occurs
Reflexivity	When Researchers examine their own biases and make them known

Reliability is defined as the development of rules and procedures contained in the protocols which enhance the reliability of case study research (Yin, 1994). Guba and Lincoln (2005) argue that as the Researcher presents increasing understanding of the phenomenon through thick description, it is up to the readers to transfer their understanding to other contexts and assess the similarity. The Researcher accepts that her constructivist world view, values and dispositions have an influence on the knowledge she has constructed in the process of inquiry. The process of transcription of video is highly iterative. Revisions of transcripts multiple times "eventually provide a reliable record of what the Researchers view as the most relevant aspects of the video for their research questions" (Engle, Conant & Greeno, 2007; Mischeler, 1991). In the Focus Group meeting the four teachers in the study were given a list of the strategies observed by the Researcher from the teachers' classroom practice. All of the

teachers agreed that the strategies existed and that the classification given to each strategy was credible although there was some discussion of a few classifications that could belong to more than one class. For the purpose of analysis the Researcher argued that the classifications should remain as designated and offered reasons to support this stance, which were accepted.

### **3.5 Chapter Summary**

Barron and Engle (2007) suggest that “an explicit multi-stage analytic approach can strengthen the likelihood of generating strong findings that are both reliable and valid” (p. 33). The purpose of this chapter was to give the reasons for the methodological approach taken in the study. Each of the methods used in this mixed methods study were explained and justified. The design of the research, including the data-gathering instruments employed to gather both quantitative and qualitative data, were described. Finally the validity and reliability of the data gathered was discussed. The use of mixed methods to collect data increased the level of confidence in the research findings. The use of multiple sources of data in the study contributed to internal validity of the work and provided triangulation (Patton, 1990). Multiple cycles of analysis also contributed to the quality of the research findings. The next two chapters contain the results of the study.

# Teachers' Beliefs and Practice

## 4.1 Introduction

In this chapter the results from the use of the instruments Questionnaire I (Appendix I) and II (Appendix 2) are analysed and summarized. The data are analysed to provide support to the qualitative data, which formed the major research component of this study. The results and analysis of the qualitative data are described in the next chapter. In the design of the study the use of the instruments Questionnaire I and II was an attempt to quantify and compare the complexity of the teachers' beliefs about mathematics, teaching and learning, and to gain insight into their practice. The research questions in the study are:

- 1 What were the teachers' beliefs about the teaching and learning of algebra?
  - (a) How did they view the educational importance of algebra?
  - (b) How did they consider the teaching of number?
  - (c) How did they choose to balance transmission, connectionist and discovery strategies to teach beginning algebra?
- 2 How does reflection upon these particular beliefs and practices inform the teachers' methods for fostering the successful conceptual development of key beginning algebraic ideas?

The research questions address specifically the teacher's beliefs about the teaching and learning of algebra. Given the nature of the first question, three sub questions were created to further explore the teachers' beliefs and practice. The instruments used had been produced for a much larger study in England to investigate teachers' beliefs and practice in teaching algebra (Swan, 2005). In the large English study three independent methods were used to ensure the validity and reliability of the beliefs and practices questionnaire. Pen portraits of a small sample of teachers, lesson observations and student questionnaires were used for this purpose.

There are seven main sections in this chapter. Section 4.1 provides the introduction to the chapter. Section 4.2 provides a description of the sample. Sections 4.3, 4.4 and 4.5 report the analysis of the teachers' beliefs about the nature of mathematics, learning and teaching respectively. Section 4.6 reports the analysis of the teachers' practice. Section 4.7 provides a conclusion to the chapter and Section 4.8 provides a summary of the findings.

## **4.2 Sample Information**

For the purposes of maintaining anonymity teachers were assigned a pseudonym.

Table 7 Teacher pseudonyms

Teacher	Pseudonym
1	John
2	Grace
3	Penny
4	Marie

The four teachers were teachers from independent schools in Western Australia. Questionnaires I and II were given to the teachers at the beginning of the study.

## **4.3 Beliefs about the Nature of Mathematics**

The data collected from Questionnaire I were entered into an Excel spreadsheet. Table 8 shows the weightings that each teacher assigned to each orientation within the category of beliefs about the nature of mathematics.

Table 8 The teachers' professed beliefs about the nature of mathematics

Teacher	Transmission	Discovery	Connectionist
John	40	25	35
Grace	10	70	20
Penny	20	30	50
Marie	40	30	30

The data was then graphed so that a comparison of the teachers' beliefs could be made.

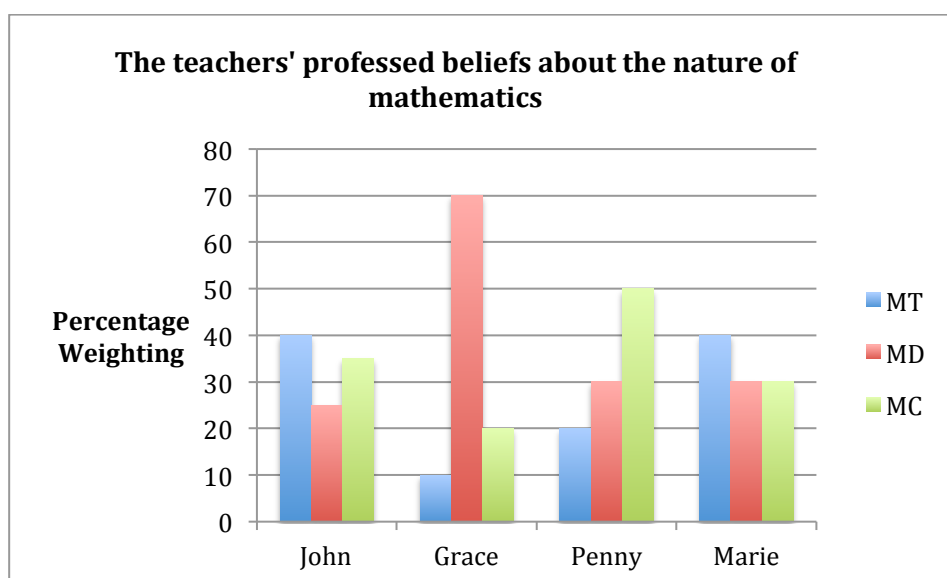


Figure 8 Comparison of teacher's professed beliefs about the nature of mathematics

MT: A body of knowledge and standard procedures. A set of universal truths and rules which need to be conveyed to students.

MD: A creative subject in which the teacher should take a facilitating role, allowing students to create their own concepts and methods.

MC: An interconnected body of ideas which the teacher and the student create together through discussion.

Just how much professed beliefs may determine or affect teachers' practice is explored in Questionnaire II. While there are clear differences between the weightings which the teachers awarded to each orientation there are some areas of similarity. John and Marie



gave their highest weighting to the belief that mathematics is a body of knowledge and standard procedures. The salient difference is the large weighting given by Teacher Grace to the belief that mathematics is a creative subject that should be learned through discovery.

Table 9 Summary of the teachers' primary beliefs about the nature of mathematics

John	Grace	Penny	Marie
Primary Belief	Primary Belief	Primary Belief	Primary Belief
a body of knowledge and standard procedures and a set of universal truths and rules, which need to be conveyed to students	a creative subject in which the teacher should take a facilitating role, allowing students to create their own concepts and methods.	an interconnected body of ideas, which the teacher and the student create together through discussion.	a body of knowledge and standard procedures and a set of universal truths and rules, which need to be conveyed to students.
Orientation-Transmission	Orientation-Discovery	Orientation-Connectionist	Orientation-Transmission

Although each teacher gave one perspective on the nature of Mathematics a higher weighting than the others (primary beliefs) it should be noted that all teachers gave some weighting to all three perspectives.

#### 4.4 Beliefs about the Nature of Learning

The data collected from Questionnaire I were also entered into an Excel spreadsheet. The table below shows the weightings that each teacher assigned to each orientation within the category of beliefs about the nature of learning.

Table 10 The teachers' professed beliefs about the nature of learning

Teacher	Transmission	Discovery	Connectionist
John	20	40	40
Grace	20	60	20
Penny	10	20	70
Marie	20	30	50

The data was then graphed for comparison to be made of the teachers' beliefs.

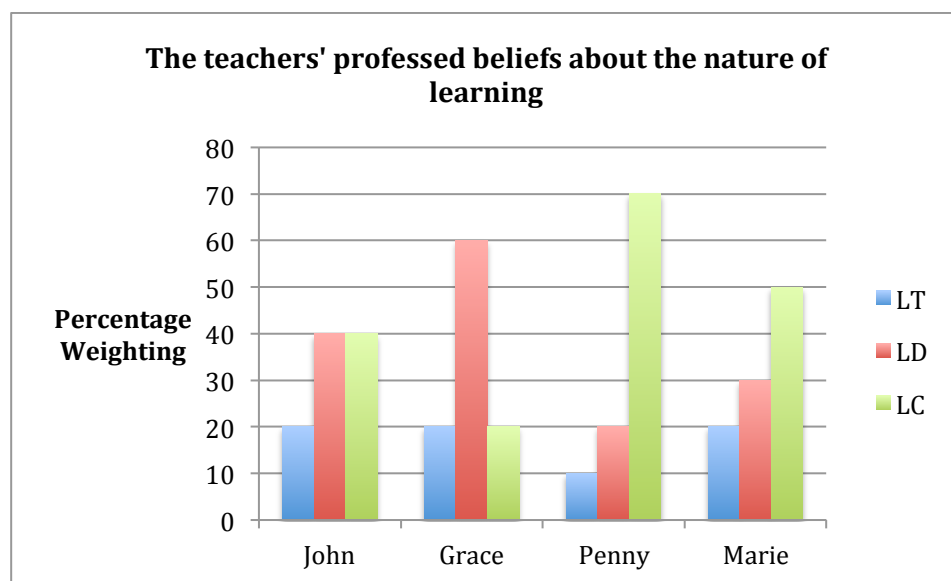


Figure 9 Comparison of teacher's professed beliefs about the nature of learning

LT: An individual activity based on watching, listening and imitating until fluency is attained.

LD: An individual activity based on practical exploration and reflection.

LC: An interpersonal activity in which students are challenged and arrive at understanding through discussion.

John awarded equal weighting to two of the three orientations. This would suggest that John believes that students learn equally well both on their own and working with others. Penny and Marie believed learning is predominately achieved when working with others in discussion. Grace awarded the largest weighting to the belief that learning is primarily an individual pursuit. Just how much this may determine or affect the teachers' practice is explored by Questionnaire II.

Table 11 Summary of the teachers' primary beliefs about the nature of learning

John	Grace	Penny	Marie
Primary Beliefs	Primary Belief	Primary Belief	Primary Belief
an individual activity based on practical exploration and reflection	an individual activity based on practical exploration and reflection.	an interpersonal activity in which students are challenged and arrive at understanding through discussion	an interpersonal activity in which students are challenged and arrive at understanding through discussion
an interpersonal activity in which students are challenged and arrive at understanding through discussion.			
Orientation-Discovery Connectionist	Orientation-Discovery	Orientation-Connectionist	Orientation-Connectionist

#### 4.5 Beliefs about the Nature of Teaching

Lastly the data collected from Questionnaire I were entered into an excel spreadsheet. The table below shows the weightings that each teacher assigned to each orientation within the category of beliefs about the nature of teaching.

Table 12 The teachers' professed beliefs about the nature of teaching

Teacher	Transmission	Discovery	Connectionist
John	30	35	35
Grace	20	65	15
Penny	20	40	40
Marie	15	60	25

The data were then graphed for comparison to be made of the teachers' beliefs.

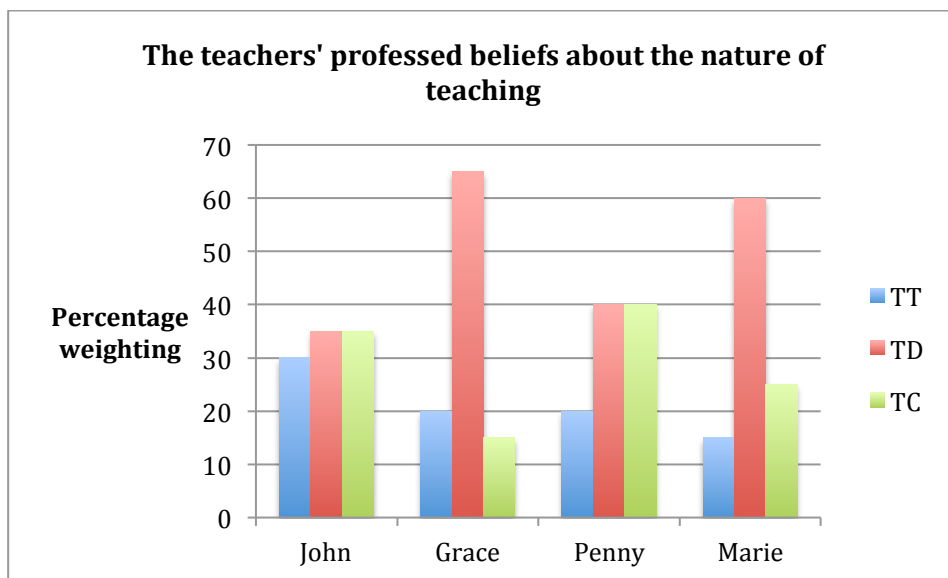


Figure 10 Comparison of teachers' beliefs about the nature of teaching

TT: Structuring a linear curriculum for the students; giving verbal explanations and checking that these have been understood through practice questions; correcting misunderstandings when students fail to 'grasp' what is taught.

TD: Assessing when a student is ready to learn; providing a stimulating environment to facilitate exploration; avoiding misunderstandings by the careful sequencing of experiences.

TC: A non-linear dialogue between teacher and student in which meanings and connections are explored verbally. Misunderstandings are made explicit and worked on.

John and Penny awarded equal weighting to two of the three approaches listed. This would suggest that John and Penny believe teaching's primary purpose is to facilitate discovery learning by structured inquiry. However the difference in the weightings given by these two teachers indicates that John believes transmission strategies are almost equally as important. All teachers in the study have given some weight to transmission as a belief about what teaching is, but John has awarded the largest weighting to this belief. Grace and Marie indicate that a discovery or inquiry belief paradigm guides their teaching. Just how much these beliefs may determine the teachers' practice is explored in the use of Questionnaire II.

Table 13 Summary of the teacher's primary beliefs about the nature of teaching

John	Grace	Penny	Marie
Primary Beliefs	Primary Beliefs	Primary Beliefs	Primary Beliefs
Assessing when a student is ready to learn; providing a stimulating environment to facilitate exploration; avoiding misunderstandings by the careful sequencing of experiences.	Assessing when a student is ready to learn; providing a stimulating environment to facilitate exploration; avoiding misunderstandings by the careful sequencing of experiences	Assessing when a student is ready to learn; providing a stimulating environment to facilitate exploration; avoiding misunderstandings by the careful sequencing of experiences.	Assessing when a student is ready to learn; providing a stimulating environment to facilitate exploration; avoiding misunderstandings by the careful sequencing of experiences
A non-linear dialogue between teacher and student in which meanings and connections are explored verbally. Misunderstandings are made explicit and worked on		A non-linear dialogue between teacher and student in which meanings and connections are explored verbally. Misunderstandings are made explicit and worked on	
Orientation-Discovery Connectionist	Orientation-Discovery	Orientation-Discovery Connectionist	Orientation-Discovery

Table 14 Summary of quantitative data for the four teachers collected from Questionnaire I

Belief	John	Grace	Penny	Marie	Mean %	SD <sup>a</sup>
Mathematics is.....						
Transmission	40	10	20	40	28	15
Discovery	25	70	30	30	39	21
Connectionist	35	20	50	30	39	13
Learning is.....						
Transmission	20	20	10	20	18	5
Discovery	40	60	20	30	38	17
Connectionist	40	20	70	50	45	21
Teaching is.....						
Transmission	30	20	20	15	21	6
Discovery	35	65	40	60	50	15
Connectionist	35	15	40	25	29	11

Note.<sup>a</sup> For mean and standard deviation measures: Percentages have been rounded to whole numbers

From the data displayed in Table 14 that explore beliefs about learning and teaching the smallest means and standard deviations are for the transmission orientation. This finding suggests that the teachers in this study do not believe that a strong orientation to transmission strategies is the preferred way for teachers to teach or students to learn.

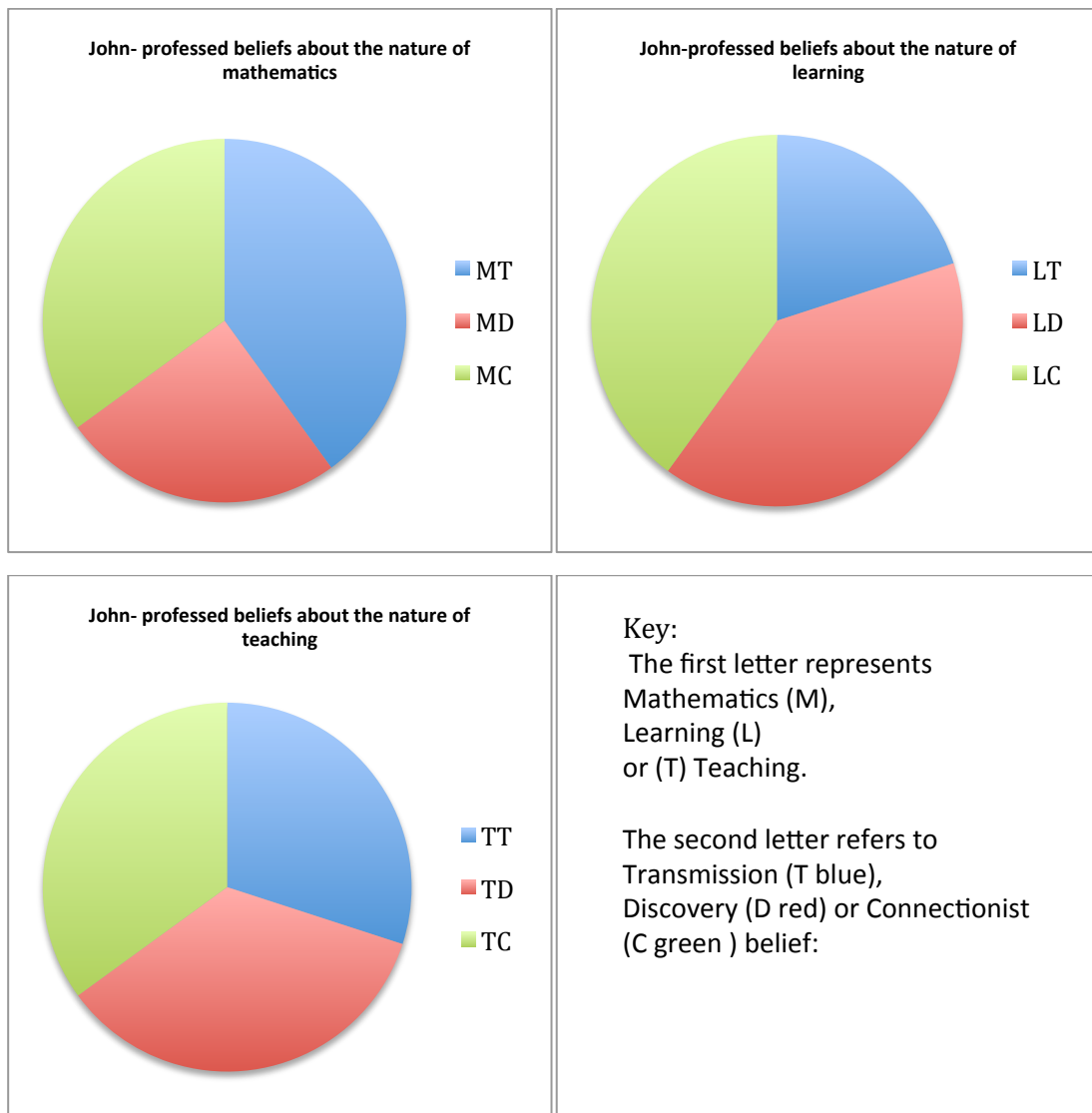


Figure 11 Summary of John's beliefs

#### Summary of results from questionnaire I: John.

- John believes that the nature of teaching can be explained by drawing from each of the transmission, discovery and connectionist orientations in almost equal measure.
- John believes that the nature of learning can be explained more from the discovery and connectionist orientations than by transmission.
- John believes that the nature of mathematics is to be explained by drawing from each of the orientations: transmission, discovery and connectionist; transmission being the most dominant orientation, followed closely by discovery.

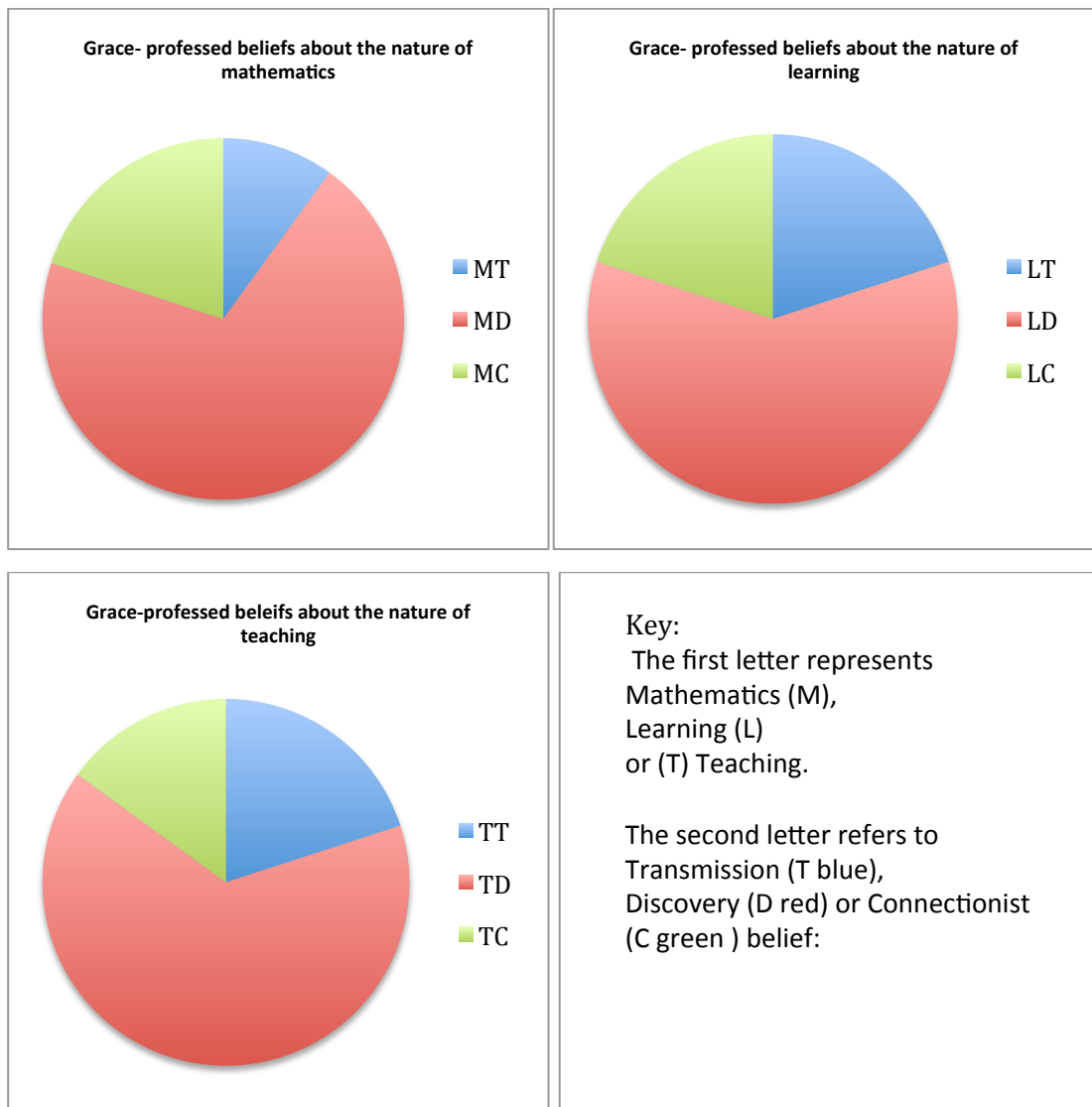


Figure 12 Summary of Grace's beliefs

### Summary of results from questionnaire I: Grace

- Grace believes that the nature of teaching can be explained primarily from a discovery orientation. Teaching is facilitating students' exploration and being aware of students' learning needs.
- Grace believes that the nature of learning can be explained by drawing from each of the orientations: transmission, discovery and connectionist. Transmission is the most prevalent orientation. Listening and imitating are important.
- Grace believes that the nature of mathematics is to be explained primarily from a discovery orientation. It is a creative subject in which the teacher should take a facilitating role.



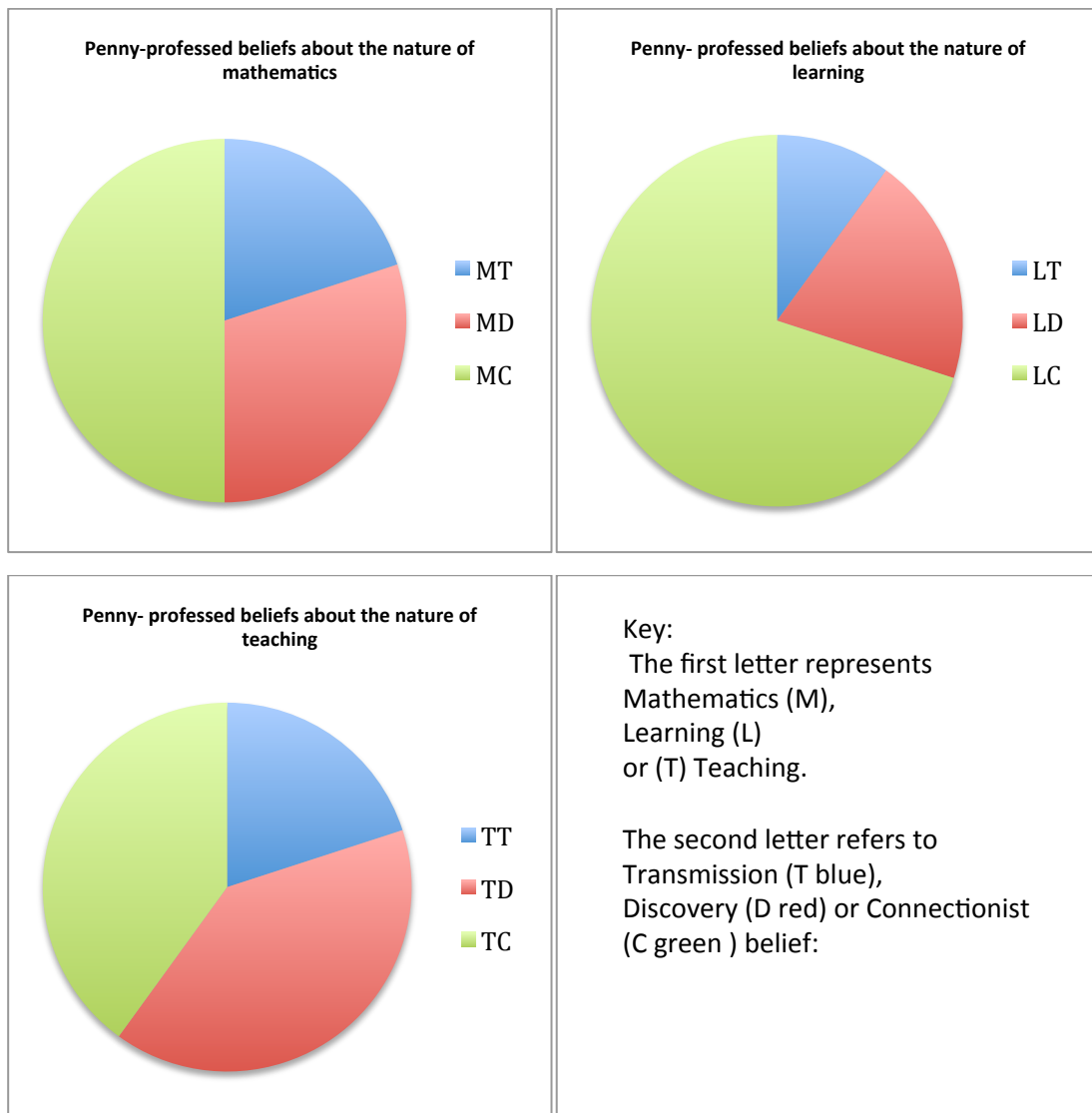


Figure 13 Summary of Penny's beliefs

#### Summary of results from questionnaire I: Penny

- Penny believes that the nature of teaching can be explained more from the discovery and connectionist orientations than by transmission.
- Penny believes that the nature of learning can be explained primarily from a connectionist orientation. Students are challenged and arrive at understanding through discussion.
- Penny believes that the nature of mathematics is to be explained primarily from a connectionist orientation. It is a network of ideas created by students and teacher through discussion

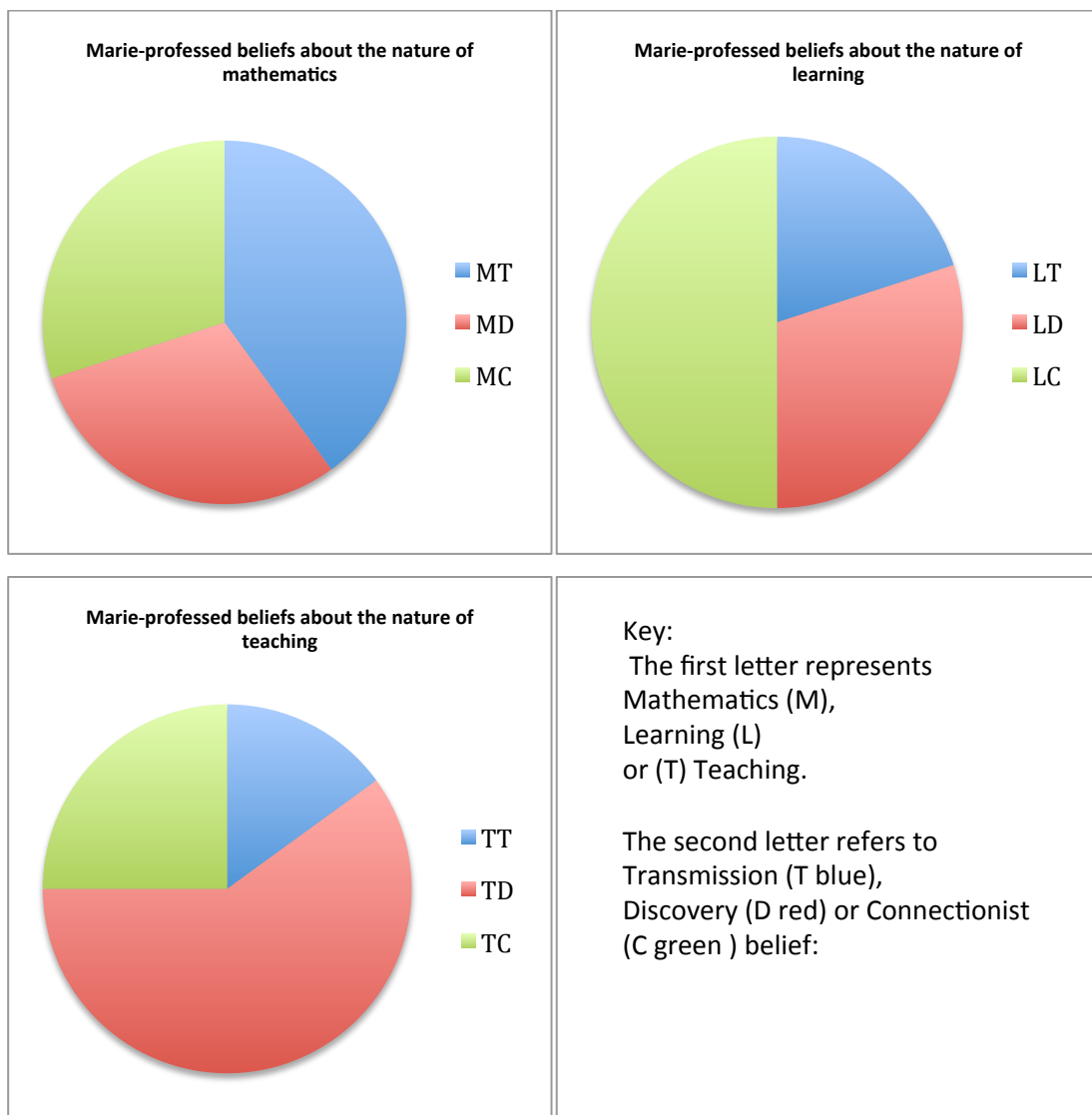


Figure 14 Summary of Marie's beliefs

#### Summary of results from questionnaire I: Marie

- Marie believes that the nature of teaching can be explained primarily from the orientation of discovery. Teaching is facilitating students' exploration and being aware of their learning needs.
- Marie believes that the nature of learning can be explained primarily from the orientation of connectionist than and least from the orientation of transmission.
- Marie believes that the nature of mathematics is to be explained by drawing from each of the transmission, discovery and connectionist orientations in almost equal measure.

## 4.6 The Teachers' Practice

In this section the teachers' favoured practice is compared with their beliefs. Teachers were asked to rate 25 statements in terms of how often they occurred in their classrooms. The five point scale ranged from almost never (1) up to almost always (5). The statements are recorded in two separate tables.

Table 15 Data collected from Questionnaire II on teacher practice-teacher centred

Statements	John	Grace	Penny	Marie
Students learn through doing exercises	3	3	4	4
Student's work on their own, consulting a neighbour from time to time	3	3	3	2
Students use only the methods I teach them	2	4	3	4
Students start with easy questions and work up to the harder questions	2	4	4	4
I teach each topic from the beginning, assuming they know nothing	2	3	3	4
I teach the whole class at once	3	5	4	3
I try to cover everything in a topic	3	4	3	3
I avoid students making mistakes by explaining things carefully first	4	4	3	4
I tend to follow the text or the workbooks closely	2	3	3	3
I know exactly what maths the lesson will contain	4	4	4	4
I tell students which questions to tackle	4	4	3	3
I only go through one method for doing each question	1	3	3	1
I tend to teach each topic separately	4	4	4	2

Code: (5) almost always, (4) most of the time, (3) half the time, (2) occasionally, (1) almost never.

Table 16 Data collected from Questionnaire II on teacher practice-student centred

Statements	John	Grace	Penny	Marie
Students choose which questions they tackle	1	2	3	2
I encourage students to work more slowly	2	3	2	4
Students compare different methods for doing questions	4	3	2	2
I draw links between topics and move back and forth between topics	4	4	3	4
I am surprised by the ideas that come up in a lesson	3	2	2	2
Students learn through discussing their ideas	5	4	4	4
Students work collaboratively in pairs or small groups	4	5	3	3
Students invent their own methods	2	4	2	2
I encourage students to make and discuss mistakes	5	4	4	2
I find out which parts students already understand and do not teach those parts	3	4	3	2
I teach each student differently according to individual needs	4	2	2	2
I jump between topics as the need arises	2	2	2	2

Code: (5) almost always, (4) most of the time, (3) half the time, (2) occasionally, (1) almost never.

From these data the following can be said about each teachers' beliefs about their practice.

John believes that most of the time in his teaching he:

- knows exactly what the maths lesson will contain;
- tells students which questions to tackle;
- teaches each topic separately;
- explains things carefully in the first instance to prevent students from making mistakes;
- draws links between topics and moves back and forth between, and
- allows students to compare different methods for doing questions.

John encourages students to make and discuss mistakes. John believes that students learn through discussing their ideas.

Grace believes that most of the time in her teaching she:

- tries to cover everything in the topic;
- knows exactly what the maths lesson will contain;
- tells students which questions to tackle;
- teaches each topic separately;
- gives students easy questions to start with and works up to the harder ones;
- expects students to use only the methods she teaches them;
- explains things carefully in the first instance to prevent students from making mistakes;
- finds out what students already know and does not teach these parts;
- allows students to invent their own methods, and
- draws links between topics and moves back and forth between topics.

Grace's students always work collaboratively and she believes that students learn through discussing their ideas. Grace always teaches the whole class at once.

Penny believes that most of the time in her teaching that she:

- teaches each topic separately, and
- knows exactly what maths the lesson will contain.

Penny believes that students learn through discussing ideas and doing exercises. She encourages her students to make and discuss mistakes.

Marie believes that most of the time in her teaching that she:

- encourages students to work more slowly;
- expects students to use only the methods she teaches them;
- gives students easy questions to start with and works up to the harder ones;
- teaches each topic from the beginning assuming the students know nothing, and



Table 17 Statistical analysis of the teachers' responses to questions related to student centred practice

Student Centred Practice	Mean	SD
Students learn through discussing their ideas	4.25	0.50
Students work collaboratively in pairs or small groups	3.75	0.96
I encourage students to make and discuss mistakes	3.75	1.26
I draw links between topics and move back and forth between topics	3.75	0.50
I find out which parts students already understand and don't teach those parts	3.00	0.82
Students compare different methods for doing questions	2.75	0.96
I encourage students to work more slowly	2.75	0.96
Students invent their own methods	2.50	1.00
I teach each student differently according to individual needs	2.50	1.00
I am surprised by the ideas that come up in a lesson	2.25	0.50
Students choose which questions they tackle	2.00	0.82
I jump between topics as the need arises	2.00	0.00

Code: (5) almost always; (4) most of the time; (3) half the time; (2) occasionally; (1) almost never.

The highest means found in this data show that the teachers believe that students should work together to discuss ideas. The statements with a mean of 2.5 or below suggest that the teachers did not generally teach individual students differently or allow students to choose the questions they tackled. They adhered to a teaching plan rather than change topic to suit student needs as they arose.

The greatest variation in practice between the teachers related to encouraging students to make and discuss mistakes. This is demonstrated by the highest standard deviation of 1.5.

Table 18 Statistical analysis of the teachers' responses to questions related to teacher centred practice

Teacher Centred Practice	Mean	SD
I know exactly what maths the lesson will contain	4.00	0.00
I teach the whole class at once	3.75	0.96
I avoid making mistakes by explaining things carefully first	3.75	0.50
Students start with easy questions and work up to the harder questions	3.50	1.00
I tend to teach each topic separately	3.50	1.00
Students learn through doing exercises	3.50	0.58
I tell students which questions to tackle	3.50	0.58
I try to cover everything in a topic	3.25	0.50
Students use only methods I teach them	3.25	0.96
I teach each topic from the beginning, assuming they know nothing	3.00	0.82
Students work on their own, consulting a neighbour from time to time	2.75	0.50
I tend to follow the text book or work sheets closely	2.75	0.50
I only go through one method for doing each question	2.00	1.15

Code: (5) almost always; (4) most of the time; (3) half the time; (2) occasionally; (1) almost never.

The highest three means found in this data show that the teachers generally taught the whole class at once, knowing exactly what the lesson would contain, and set the students easy questions before progressing to the more difficult. The statement with the lowest mean suggest that the teachers used more than one method to teach students how they might find the solution to a problem.



#### **4.7 Summary of the Teachers' Professed Beliefs about the Nature of Mathematics, Teaching and Learning.**

In the following summaries the word 'believes' refers to the teachers' responses to the questionnaire items. John believes that mathematics teaching involves employing approaches which stem from all three orientations. He believes that learning is student centred and the teacher is facilitator. The nature of mathematics is to be explained by drawing from each of the transmission, discovery and connectionist orientations with transmission being the most dominant orientation, followed closely by discovery.

Grace believes that the nature of teaching can be explained primarily from a discovery orientation. Teaching is facilitating students' exploration and being aware of students' learning needs. The nature of learning can be explained by drawing from each of the transmission, discovery and connectionist orientations, with transmission the most prevalent orientation. Listening and imitating are important. The nature of mathematics is to be explained primarily from a discovery orientation. It is a creative subject in which the teacher should take a facilitating role.

Penny believes that the nature of teaching can be explained more from the orientations of discovery and connectionist than by transmission. The nature of learning can be explained primarily from a connectionist orientation. Students are challenged and arrive at understanding through discussion. The nature of mathematics is to be explained primarily from a connectionist orientation. It is a network of ideas created by students and teacher through discussion.

Marie believes that the nature of teaching can be explained primarily from the orientation of discovery. Teaching is facilitating students' exploration and being aware of students' learning needs. The nature of learning can be explained primarily from the orientation of connectionist and least from the orientation of transmission. The nature of mathematics is to be explained by drawing from each of the transmission, discovery and connectionist orientations in almost equal measure.

Both Grace and Marie believe that the nature of teaching is best described from a discovery orientation. Penny believes that both discovery and connectionist orientations best

describe the nature of teaching. John believes a balanced approach emanating from all three orientations is the nature of teaching.

#### **4.8 Summary of the Teachers' Practices**

The results from the questionnaire show that John claimed to use more student centred practices than teacher centred practice most of the time. Marie, Grace and Penny used more teacher centred practices than student centred practices most of the time. However all four teachers on the practices scale demonstrate a 'middle ground'. In summary, this chapter has reported the chief findings of the quantitative data analysis phase of this study. In the next chapter the findings of the qualitative data are presented.

# Teachers' Beliefs, Practice and Reflection on Practice

## 5.1 Introduction

This chapter has four sections. Section 5.2 presents a summary of the research design including the processes used to collect and analyse the interviews, plenary discussion and video data. Section 5.3 describes the analysis of the data. In Section 5.4 an overview of the data is presented. The qualitative data from this chapter is interwoven with the quantitative data in the individual case studies which will be further outlined in Chapter Six. The combination of data from different sources is used to draw rich descriptions of each of the teachers' professed beliefs and practices in an attempt to answer the two research questions of this study.

The research questions in the study are:

1. What were the teachers' beliefs about the teaching and learning of algebra?
  - (a) How did they view the educational importance of algebra?
  - (b) How did they consider the teaching of number?
  - (c) How did they choose to balance transmission, connectionist and discovery strategies to teach beginning algebra?
2. How does reflection upon these particular beliefs and practices inform the teachers' methods for fostering the successful conceptual development of key beginning algebraic ideas?

The validity and reliability of the findings are enhanced by the triangulation of the data with continual comparison and revisions being made (Thomas, 2003; Punch, 1998).

## 5.2 Overview of Design

Each teacher was interviewed at the beginning and at the end of the data-gathering phase of the study. Interview dates and times were negotiated with each of the teachers and interviews took place at the different schools. The interviews, whose duration varied, were recorded and notes were made during the interviews. The taped interviews were then transcribed with the assistance of Dragon V3 software (Nuance, 2012). The Researcher was aware of the danger of bias or influence in the semi-structured interviews, and therefore interview protocols were put in place. The questions asked at the first interview were designed to yield data that would inform Research Question One, and in particular part (a) of the question,

1. (a) How did the teacher view the educational importance of algebra?

The questions for Interview 1 were intended to determine if possible the teacher's background as a teacher of algebra, and their beliefs about the teaching and learning of algebra. The first set of interview questions (Interview 1) were as follows:

1. Approximately how many years have you been teaching?
2. Of this time, how much of it has been spent teaching algebra?
3. What is your experience of teaching algebra to Year Seven students?
4. How do you feel about teaching algebra?
5. Would you say that algebra is an important strand of mathematics?  
Please give reasons for your answer.
6. Are there any particular hurdles, in the study of algebra, which you have observed students experience in the early work in algebra?
7. In your opinion, what would be the major causes of problems in learning algebraic strategies and techniques?

The questions asked at the second interview were designed to yield data that would reveal more about Research Question One and in particular part (a) of the question, and also to yield data for Research Question Two.

The second set of Interview questions (Interview 2) were as follows:

1. Did you find what you expected to find in terms of the outcomes of student learning of algebra at the end of the unit?
2. If you found any changes, what were these, and can you suggest why this may have been the case?
3. At the completion of the unit of work on algebra how would you describe your level of confidence in teaching this subject matter? Please give reasons for your answer.

The focus group meeting held at the AISWA offices on the 5<sup>th</sup> of November 2012 was designed to be both a professional development opportunity for participants and a means to gather further data for the researcher. The aim of the professional development was to provide the teachers with information about the theory behind the study.

During the meeting it was suggested that practice involves using strategies from the three orientations. At this meeting the validity and reliability of aspects of the qualitative data gathered in the lesson observations were confirmed (Patton, 2002). Teachers were presented with a list of strategies observed in their algebra lessons, and asked for comment.

Table 19 Strategies from observation of practice

Strategy	Code
1	Allows students to investigate
2	Allows students to make errors
3	Asks students for reasoning
4	Calculator use
5	Lesson starter
6	Issues manipulatives
7	Choice of questions
8	Self assessment or peer assessment
9	Students make their own notes
10	Students work with others
11	Personalized learning
12	Closed questions

13	Definitions presented and explained
14	Direct instructions
15	Follows textbook
16	Identifies content at the start of the lesson
17	Identifies likely errors or confusion
18	Models procedures or process
19	Notes provided by teacher
20	Students work individually
21	Teacher assessment
22	Use of practice in class
23	Whole class instruction
24	Connection to prior learning
25	Connections made between ideas
26	Links to other subjects
27	Less is more—Time for elaboration
28	Engagement focus
29	Significant contribution to lesson

The teachers all agreed that these strategies represented their classroom practices. The qualitative data gathered from this instrument were used to contribute to the development of themes, and to address Research Questions One and Two. The data were also used to develop the case. More detailed definitions of the strategies are provided in Section 5.3.1.

At the completion of the unit of work in beginning algebra the teachers were asked to write a response to four questions. These questions asked them to reflect both on their teaching practice and their students' learning.

1. In the recent unit of work on algebra, what would you say were the most successful lessons?
2. What do you think made the difference?
3. Were there any surprises, that is, questions or insights offered by students you did not expect? If so, why do you think this happened?
4. Is there anything you would do differently? If you answer yes to this question, can you elaborate by saying what you would change and why?

The Researcher observed each teacher in the study teaching four lessons from a unit of work on beginning algebra. The lessons varied in length from 50 minutes to 80 minutes. Some teachers taught their students three times a week while others four times a week.

Table 20 Frequency of lessons and duration

Teacher	Number of Lessons per week	Length of lesson
John	4	60 minutes
Grace	3	75 minutes
Penny	4	50 minutes
Marie	3	80 minutes

Initially a small camera was used to record lessons but concerns arose over reliability and it was thus replaced by a larger camcorder. This device was used for the majority of the recording. The camera was mounted on a tripod. The Researcher was positioned at the rear of the classroom with the intention of being the least possible distraction to the students and the teacher.

### 5.3 Analysis of Data

The transcripts from Interview 1 were coded and any notes taken added to the data, which was later examined to search for any themes or patterns. It appeared in early analysis that emergent themes centred around three key ideas. Those ideas were firstly teacher knowledge, secondly number and thirdly, knowledge of algebra providing the basis of aspects of the teacher's pedagogical content knowledge (Shulman, 1986).

The focus group meeting provided a rich source of data and took place after the completion of the lesson observations. At this meeting the four teachers met for the first time. All discussions at the meeting were recorded and transcribed to deliver data for reflection, and to contribute to the building of the case. The transcribed data were then colour-coded to identify each of the teachers and then blocked to build a picture of each teacher's thoughts and ideas about the subject matter discussed. This process yielded data that contributed to the construction of individual case stories. The emergent themes from this data supported

those noticed in the first interview, namely, teacher beliefs and knowledge, number and algebra pedagogical content knowledge.

Using Artichoke software (Fetherston, 2012) each of the 16 lessons observed were transcribed and the data collected were stored in a database created in Artichoke. The Researcher then examined the video captured from each lesson for each teacher and coded selected actions of the teacher into the database. Segments of data within each lesson were designated examples of the teacher using a particular teaching strategy, as defined in Section 5.3.1. Transcripts from the database were printed and the Researcher analysed these so as to confirm or refute initial coding. Copies were made of the transcripts of each teacher's practice and any further changes were made. A final coding was then established for each lesson. These data were used to explore the composition of each teacher's lessons, and tables were constructed to display the teachers' practice. The coding was validated by the use of two different methods of coding and the reliability appears confirmed by the corresponding findings from both sets of data (Lincoln & Guba, 1985).

Data from Questionnaires I and II were reconsidered in conjunction with lesson observations, which resulted in an improved table of teaching strategies (Table 19). The initial list of strategies was refined several times before the final list of strategies was complete. This work was based on Video workflow research processes described by Pea and Hoffert (2007), which show an iterative approach to video data analysis. The data are broken into manageable sections, identified, transcribed and categorized, before analysis. Analysis may require that new categories are created and as the data are explored further it may be necessary to reconsider the first 'chunking' of data, and thus the cycle continues. The complete diagram explaining such iterative process used in the study can be found in the methods chapter of this thesis (Figure 7). The identified teaching strategies were recorded and coded in each lesson to assist the Researcher to examine the balance of strategies used by the teachers. The strategies identified in the lessons were then given a classification corresponding to those used in both Questionnaire I and II. The strategy was then identified as originating from a discovery, transmission or connectionist orientation (Askew, et al., 1997).

By using the concepts in Swan's practices Questionnaire II, 25 classroom behaviours were identified. Twelve of these behaviours were defined as teacher-centred and 13 were defined as student-centred. In the current study this idea was further developed. The



particular belief orientation about mathematics learning and teaching gathered from the Questionnaires was linked with the identified strategies the teachers used to achieve their lesson objectives. What follows is a detailed list of the strategies used, and a definition given to justify the classification given to the strategy. It could be argued that the identified strategies could have been classified somewhat differently, and indeed there was discussion of instances of this at the focus group meeting. Participants suggested there were some strategies that could be defined in two or more orientations. While listening to these viewpoints, the Researcher explained the context in which the strategy had been observed in the lessons and the reason for the orientation chosen. Strategies were in fact classified in this manner because this was how the Researcher, as an experienced teacher, observed each of the strategies being used in each classroom in particular contexts. Therefore these classifications are not by any means exhaustive, as they are grounded in observation of a particular cohort, but they proved to be a useful tool for this particular study. It is also felt that the strategies would likely provide a good working model for most classroom algebra teaching practice.

### **5.3.1 Definition of strategies**

In the following definitions, (D) indicates that a strategy has been classified as discovery, (T) indicates it has been classified as transmission and (C) indicates the strategy has been classified as Connectionist.

#### **Strategy 1: Allows students to investigate (D)**

Definition: The teacher allowed the student time to come to his or her own conclusions about an idea. This investigation may or may not have been with concrete material.

#### **Strategy 2: Allows students to make errors (D)**

Definition: The student was explicitly given the opportunity to try and was told not to worry if a mistake was made.

### **Strategy 3: Asks students for reasoning (D)**

Definition: When students were asked for their reasons for giving a particular answer the teacher listened while the student described her or his thinking. The teacher supported the students to come to conclusions for themselves. The teachers were helping students to think and consolidate their ideas, to make sense for themselves.

### **Strategy 4: Calculator use for checking and exploration (D)**

Definition: This is a strategy which could quite easily be acceptable in any of the three approaches. However, the choice was made by the Researcher to assign it as a discovery approach, because in all the lessons observed, the calculator was used mainly as a tool for students to check their answers or to try something out. It was interesting that students using a calculator did not find their limited number skills prohibitive when investigating possible solutions.

### **Strategy 5: Lesson starter (D) (T)**

Definition: Two kinds of lesson starters were observed. One was defined as discovery and the other transmission. In the discovery starter the teacher used a game or a puzzle to begin the lesson. Students were asked to find something out or just to play a game; the real purpose would be discussed after the game had been played. In the transmission starter the teacher would ask the students a question or series of questions about work they had just completed, to remind them of what they had learned or to allow them to practise new skills.

### **Strategy 6: Issues manipulatives for exploration (D)**

Definition: Students were given materials like matchsticks or unit cubes to work with in a lesson. They were given time to 'play' with the equipment

before the teacher asked questions of the students. The type of questions the teacher asked required the student to discover an answer by using the objects.

#### **Strategy 7: Choice of questions (D)**

Definition: Students were asked to choose questions from an exercise in the text.

#### **Strategy 8: Non-Teacher Assessment (D)**

Definition: Students checked for themselves that their answer was correct. The teacher played no part in assessment at this point.

#### **Strategy 9: Students make their own notes (D)**

Definition: The student decided what was important at the time in regard to what to note in their notebooks or journals. The teacher may well have offered advice but it was up to the student as to what was written. Examples from student notes are provided in Appendix 9 of this thesis.

#### **Strategy 10: Students work with others (D)**

Definition: Students worked collaboratively on an open task.

#### **Strategy 11: Personalised learning (D)**

Definition: The teacher worked with the student. This was assigned work to suit the students' learning at that particular point in time.

#### **Strategy 12: Closed questions (T)**

Definition: The teacher's use of this strategy was founded in the expectation that the answer given by the student would be either right or wrong. The teacher knew the answer before it was given.

**Strategy 13: Definitions presented and explained (T)**

Definition: The teacher defined each new word and gave examples in context.

**Strategy 14: Direct instructions (T)**

Definition: The teacher gave the class explicit directions.

**Strategy 15: Follows textbook and/or worksheets (D)**

Definition: The teacher worked through the chapter from the text or worksheet in a particular order of concepts.

**Strategy 16: Identifies the lesson objectives at the start of the lesson (T)**

Definition: The objectives of the lesson were written on the whiteboard; alternatively the teacher outlined the lesson, stating clearly what he or she intended to teach.

**Strategy 17: Identifies likely errors or confusion (T)**

Definition: The teacher when modeling a process of action highlighted for the students common mistakes historically made by students. Or the teacher stopped the class working and revisited the task when he or she found that there were some common errors in thinking.

**Strategy 18: Models procedure or process (T)**

Definition: Using this strategy the teacher demonstrated on the whiteboard or the Interactive Whiteboard how to set out the problem and how to solve it.

**Strategy 19: Notes provided by teacher (T)**

Definition: The teacher wrote on the board or supplied students with a worksheet, which had the notes on it that the teacher wanted the students to learn. These notes were well defined. The student played no part in their construction. The teacher decided the composition and length of the notes.

**Strategy 20: Students work individually (D)**

Definition: The teacher set students work from the textbook or a worksheet to be completed either in class or as homework.

**Strategy 21: Teacher assessment (T)**

Definition: The teacher determined whether something was right or wrong. The student played no part in the assessment.

**Strategy 22: Use of practice in class (T)**

Definition: Students practised what they had been taught.

**Strategy 23: Whole class instruction (T)**

Definition: The teacher spoke to the whole class at once and was the centre of attention. All of the 16 lessons observed involved whole class instruction.

**Strategy 24: Connection to prior learning (C )**

Definition: The teacher assisted students to see, imagine and make the connections between new thinking and their existing body of knowledge and experience. This meant asking the student to remember a previous lesson or a previous piece of work, and to bring this to mind when looking at the new concept or process.

**Strategy 25: Connection made between ideas (C)**

Definition: This strategy was used when students were given time to investigate. (They may also have manipulated objects, although this was not often the case.) After a reasonable time given to discuss, create and invent, the teacher became more involved in the students' learning. The teacher asking thought-provoking questions, refocussing key points of the lesson, listening to students verbalise their thinking and suggesting action, all assisted the students to make the connection between ideas. This strategy is probably one of the most powerful examples of a connectionist approach where *learning and teaching are equally balanced*.

**Strategy 26: Links to other subjects (C)**

Definition: The teacher asked students to think about where they had thought about an idea before in another subject. (Where was algebra used in another learning area?)

**Strategy 27: Less is more—Time for elaboration (C)**

Definition: In the use of this strategy the teacher made a significant time allowance in the lesson and provided students with time to discover or make sense of an idea or concept that the teacher wanted the student to learn.

**Strategy 28: Engagement focus—Students with algebra (C)**

Definition: The teacher ensured that every student was involved in the thinking about the problem or question. This was achieved by inclusive questioning.

**Strategy 29: Significant student contribution to lesson (C)**

Definition: The teacher emphasis was on the students' input into the lesson. Students may have been creating, constructing or questioning and they shared

with others in the class. The teacher allowed for the exchange of ideas in discussion and the teacher became both expert and guide.

In these ways, the use of lesson transcripts and subsequent coding of teacher practice in terms of the orientation behind the strategy can be seen to yield another picture of the teachers' practice. The frequency with which strategies were used by each teacher, as identified by the Researcher from lesson transcripts was calculated to provide some sense of the lesson composition. A coded phase of a lesson is provided in Appendix 8. Tables of teachers practice were compiled to create Table 21.

Table 21 Analysis of teachers' practice

	Lesson	Transmission	Connectionist	Discovery	%T	%C	%D
JOHN	1	9	3	6	50	17	33
	2	9	5	7	40	25	35
	3	8	3	8	42	16	42
	4	5	3	6	36	22	42
Total		31	14	27			
GRACE	1	9	1	6	56	6	38
	2	8	4	7	42	21	37
	3	10	3	6	52	16	32
	4	8	2	3	62	15	23
Total		35	10	22			
PENNY	1	10	2	2	72	14	14
	2	10	1	4	67	7	26
	3	8	1	5	57	7	36
	4	9	2	5	56	13	31
Total		37	6	16			
MARIE	1	10	5	9	42	21	37
	2	5	4	10	26	21	53
	3	8	4	8	40	20	40
	4	4	5	7	25	31	44
Total		27	18	34			

The table is included here because they were part of the process of refining the large amount of data gathered from video before software was used for further analysis. To assist in identifying themes, the video data was coded and then analyzed using the Cluster analysis function on the program Artichoke (Fetherston, 2012).



Cluster analysis gathers codes or text units and places them into groups based on their similarity—similarity in the case of codes relates to the way the codes are applied (or occur) at text units. This type of analysis can be used to find out which codes belong together and consequently can be used to identify common themes. It provides an alternative to using the “Explore” and “Examine” approach described above. Fetherston explains that the difference between cluster analysis and these methods is that cluster analysis relies upon mathematics to determine similarities rather than human judgment. However, ultimately, once groups of similar codes are found it is a human judgment as to whether they are actually similar (p. 51).

The following dendograph (Figure 16) was the result.

Graph to show similarities in observed strategies

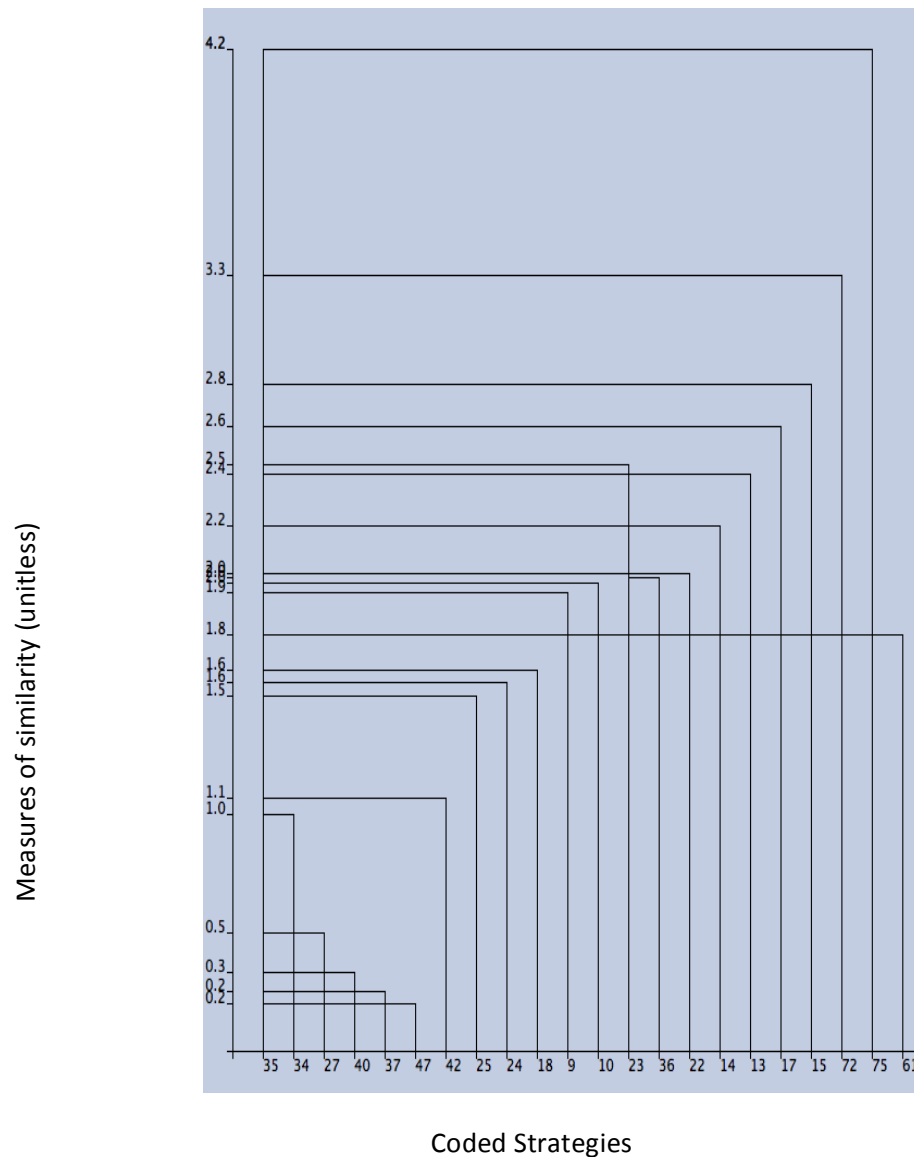


Figure 16 Dendograph 1

Key: Transmission: 15,17,22,23,36,37,40	Discovery: 18,24,25,35,34,42,47
Connectionist: 10,13,27	Teachers Codes 9,61,72,75

Numbers on the horizontal axis represent the codes corresponding to the identified strategies and the four teachers as follows: John, has code number 9; Grace, has a code number 61; Penny, has a code number 72; Marie, has a code number 75. It is interesting to note that John is some distance from Grace, Penny and Marie in this dendograph. A reason for this is suggested in Chapter Seven of the thesis. Numbers on the vertical axis are unit-less

measure of similarity. In this representation of the coded data, the smaller the number on the vertical axis the closer relationship between the strategies. From Figure 16, it is difficult to see the similar codes, so the cluster analysis was applied to the video data once again, but this time using only those codes that occurred more than 10 times for each teacher. Figure 17 shows the clustering of similar codes.

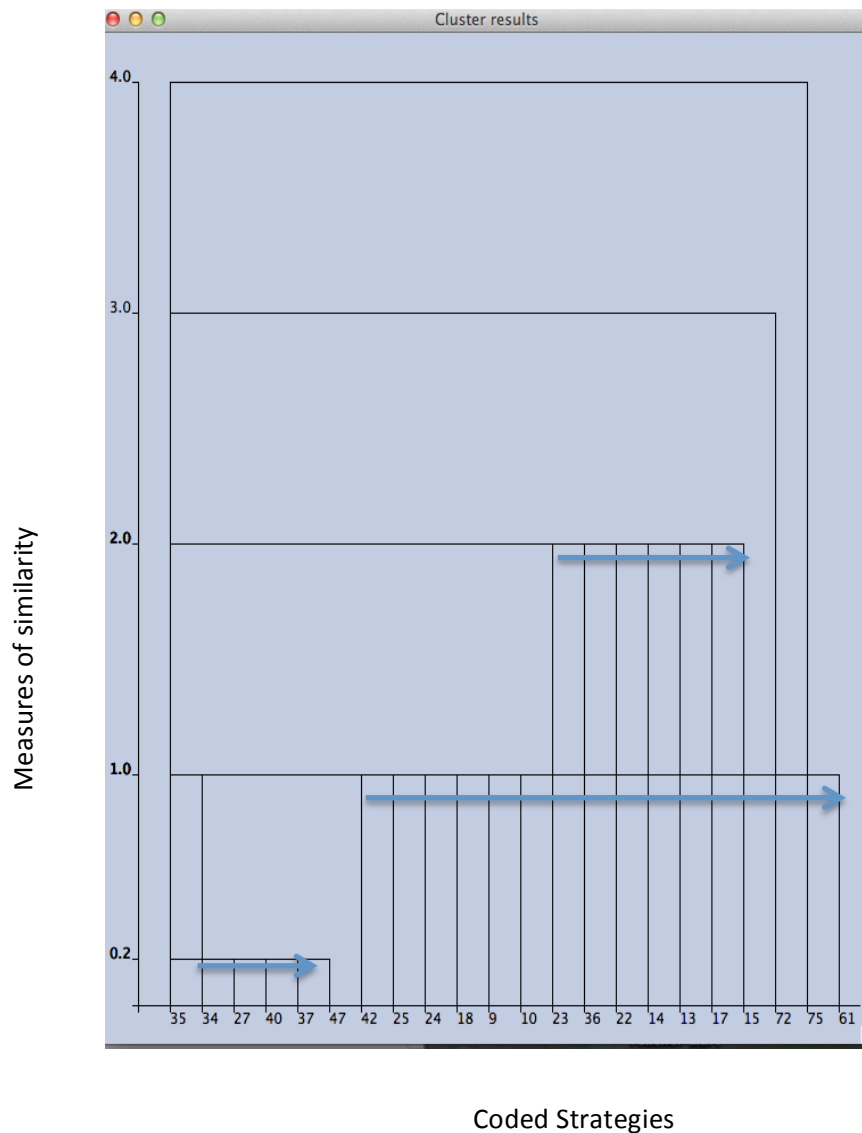


Figure 17 Dendograph 2 Cluster result

Note: the smaller the number on the vertical the more similar the code (strategy)

Key:

Transmission: 15,17,22,23,36,37,40

Discovery: 18,24,25,35,34,42,47

Connectionist: 10,13,27

Teachers Codes 9,61,72,75

The codes are grouped together as being similar using the program but there is no obvious meaning as to why this is the case. It is the task of the Researcher to look for meaning in the data. To ask the question what links these codes together in the classroom?

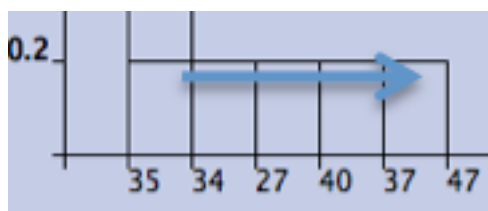


Figure 18 Teaching/Learning cluster of strategies from the dendrogram

This group of strategies link together to form a balance of transmission, discovery and connectionist strategies – a balance between teaching and learning.

Table 22 Teaching/ Learning cluster of strategies

#### TEACHING/LEARNING BALANCE

Code	Strategy	Orientation
34	Allows students to make errors	Discovery
47	Students work with others	Discovery
37	Closed questions	Transmission
40	Teacher assessment	Transmission
27	Connections made between ideas	Connectionist

Theme: Teaching/learning balance

This theme of practice contains strategies for transmission, discovery connectionist strategies identified by the researcher. There is only one connectionist strategy.

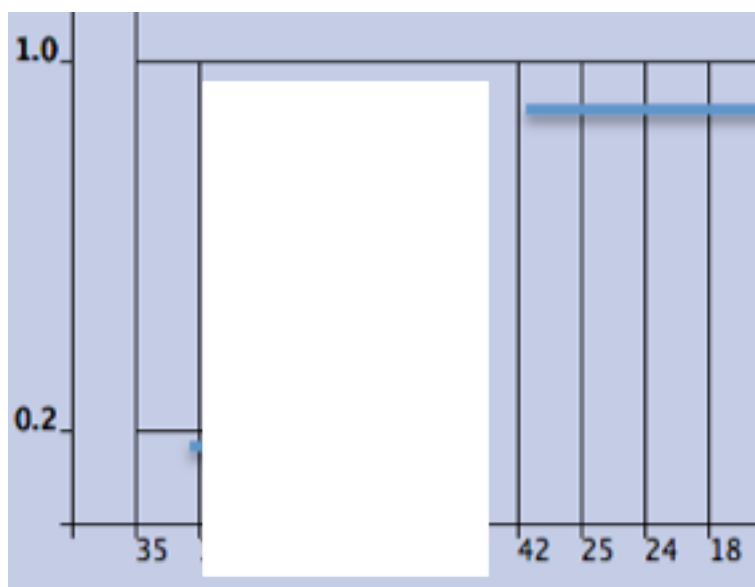


Figure 19 Teacher focus cluster of strategies from the dendograph

This group of strategies link together to form a cluster of strategies identified by the Researcher as discovery- student focus.

Table 23 Student focus cluster of strategies

**STUDENT FOCUS**

Code	Strategy	Orientation
42	Allows student to investigate	Discovery
18	Students make their own notes	Discovery
35	Personalized Learning	Discovery
24	Follows textbook, worksheets	Discovery
25	Students work individually	Discovery

Theme: Student Focus

This theme of practice contains only strategies from the discovery orientation.

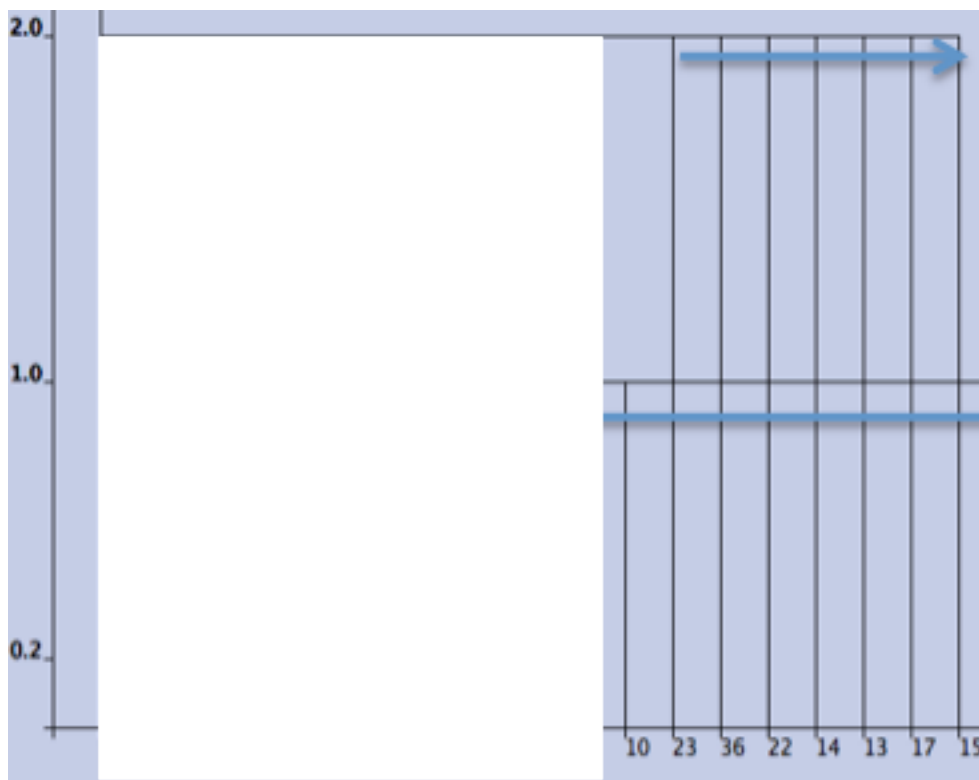


Figure 20 Teacher focus cluster of strategies and teaching/learning/student balance cluster of strategies

Table 24 Teaching/ learning/student balance cluster of strategies from the dendograph

#### TEACHING/LEARNING/STUDENT BALANCE

Code	Strategy	Orientation
10	Connections to prior learning	Connectionist
22	Models procedures and process	Transmission
14	Asks students for reasoning	Discovery
13	Engagement focus	Connectionist

#### Theme: Teaching/ Learning/Student balance

This theme of practice contains strategies for transmission, discovery connectionist strategies identified by the researcher. There are two connectionist strategies.

Table 25 Teacher focus cluster of strategies from the dendograph

#### TEACHER FOCUS

Code	Strategy	Orientation
23	Use of practice in class	Transmission
36	Direct instructions	Transmission
17	Definitions presented and explained	Transmission
15	Whole class instruction	Transmission

#### Theme: Teacher Focus

This group of strategies link together to form a cluster of strategies identified by the Researcher as transmission- teacher focus.

From the dendograph four themes were identified and classified together to form themes of practice. These four 'themes of practice' appeared to logically explain the observed teaching practice of the four teachers.

#### THEMES OF PRACTICE

- STUDENT FOCUS
- TEACHING/LEARNING BALANCE
- TEACHING/LEARNING/STUDENT BALANCE
- TEACHER FOCUS

A concern for the researcher in observing lessons was to avoid personal bias and to look at the data as it stood. Despite being a teacher of many years' experience it was important for the study that personal views were set aside and the data was looked at impersonally. Using the cluster function in Artichoke gave the researcher another more objective view of what the data was saying. The cluster analysis showed that not all strategies identified as transmission, discovery or connectionist were clustered together as three separate groups. Nevertheless, there was enough evidence to suggest that that a pattern of

themes had emerged. The researcher accepts that the application of the codes is subjective, and that in compiling the 'themes of practice' where there were strategies that formed a large cluster, the researcher, as the interpreter of the data, made the final decision on how this was disseminated (Creswell, 2008).

The transcripts from Interview 2 were analysed and these data were examined along with the qualitative data gathered from Interview 1, the focus group meeting, and the teachers' reflections using the colour coding method described on page 74. The second interview occurred in the final meeting with the teachers and was the last opportunity to find any additional data that would contribute to the developing themes or create new themes. At this stage the researcher proceeded to work through several of the stages in making sense of the qualitative data, identified by Yates (2004), as analysis, synthesis, relating to other work, and reflecting back in a cyclic fashion.

Comparisons were made between transcripts of interviews, transcripts from the focus group meeting and the teacher's reflections to look for similarities. The teacher's talked about number skills and their thoughts about algebra and how it should be taught. A synthesis of the qualitative data combined with results from the quantitative data produced the themes of:

- Knowledge and beliefs about number;
- Knowledge and beliefs about algebra;
- Knowledge and beliefs about pedagogy for teaching early algebra;
- Themes of practice, including:
  - Student focus;
  - Teaching/learning balance;
  - Teaching/learning student balance;
  - Teacher focus.

Themes of Practice emerged from the lesson observation data and these completed the themes used in the analysis of each teacher in the individual cases, which are written as narratives in Chapter Six of the thesis.



## 5.4 Chapter Summary

This chapter presented results from the qualitative data portion of the research. An analysis was provided of qualitative data gathered from several sources, namely: interviews, a focus group meeting, teacher reflections, and lesson observations. The analysis of this data supported by quantitative data gathered identified the following four themes:

- Knowledge and beliefs about number;
- Knowledge and beliefs about algebra;
- Knowledge and beliefs about pedagogy for teaching early algebra;
- Themes of practice, including:
  - Student focus;
  - Teaching/learning balance;
  - Teaching/learning/student balance;
  - Teacher focus.

Part of each of these first three themes form an aspect of the teacher's pedagogical content knowledge (Shulman, 1986), for teaching number and algebra.

In the case studies which follow, the qualitative and quantitative data gathered from each of the research instruments is used to construct each case, and through further analysis to provide answers to the research questions that guided this study.

# Case Studies

## 6.1 Introduction

In this chapter the professed beliefs and the actual practices of the four teachers in the study are explored. This exploration is consistently founded on the three underlying teaching orientations of transmission, discovery, and connectionist (Swan, 2005 Askew et al., 1997), as discussed earlier in the thesis. The discussion is therefore also necessarily related to:

- The knowledge and dispositions concerning the learning and teaching of algebra that the teachers brought with them to the classroom, and which formed the basis of their pedagogical content knowledge (PCK), (Shulman, 1986).
- The teachers' observed, individual classroom preferences as to the blend of teaching orientations, demonstrated in their choice of a range of strategies analysed in cluster analysis and later grouped in what will be called 'themes of practice'.

Three aspects of the relevant pedagogical content knowledge for the teaching of early algebra are seen in Table 26. In sections 6.2, 6.3, 6.4 and 6.5 of this chapter each of the four cases are constructed.

The following table outlines three categories of PCK (and CK) brought by the case study teachers to the classroom, and their understanding of the development of number, algebra concepts, and pedagogies for developing algebraic concepts from number concepts.

Table 26 Teacher knowledge and beliefs

The teacher's brought to classroom	Teacher knowledge and beliefs about:
<i>as evidenced in the questionnaires, case study interviews, and focus group discussions and reflections</i>	the development of number concepts
	the development of algebraic concepts
	the connection of established number (arithmetic) concepts with the effective teaching and learning of early algebra

A brief summary of each of the concepts covered by each teacher in the four observed lessons and the term in which the observations took place are to be found in table 27.

The lessons John and Penny gave were observed during term three of 2012. The lessons Grace and Marie gave were observed during term four of 2012.

Table 27 Summary of concepts covered by each of the teachers

John (Term 3)	Grace (Term 4)	Penny (Term 3)	Marie (Term 4)
Solve equations using inverse operations. The notion of balance in equations. The idea of relationship between variables. Brief look at rules for Integer operations. Find a pattern using toothpicks and find a rule to continue the pattern. Link finding the missing number problems from primary mathematics to algebra. Patterns, identify continue using shapes etc. Create a rule and substitute values. Use manipulatives to find a pattern. Complete a table of values. Plot points on the Cartesian plane, first quadrant. The draw a straight line. Extrapolate values from the line. Create a table of values and find the rule.	Introduction of algebraic terms and their definitions. Translation of a word problem into an algebraic expression or equation. Substitution into an equation. Use formulae to find area and perimeter of plane shapes. Create tables of values-linear. Use rules of the form $y=mx+b$ Write an equation from a word problem. Then solve the problem. Create flowcharts to solve an equation. Find the rule, using algebraic terms, for a pattern. Create a table	Introduction of algebraic terms and their definitions. Practice identifying terms, expressions. Practice using algebraic notation. Like terms. Translation of a phrase into an algebraic expression. Operations with like terms.	Laws of arithmetic- Associative, Commutative Distributive Expansion and factoring of algebraic terms. Simplify algebraic expressions Definitions of algebraic language coefficient, expressions etc. Difference between expressions and equations. Like terms Match word sentences or phrases with equations or expressions. Create equations. Language of operations encountered in word problems. Solve equations

There were difference and similarities in the content the teachers covered. All four teachers included translating a phrase or a worded problem in an equation in their lesson content. They spent time on the language of algebra and of problems. John and Grace covered the most common content of the four teachers. They asked students to create or find a rule to continue a pattern and to describe the relationship between two variables. Only Marie had the laws of arithmetic in her lesson content. Grace, Penny and Marie spent time introducing algebraic terms and notation. Only John included the Cartesian plane in his lesson content.

How to deal with negative integers arose in classroom discussions but there was no significant classroom content of this covered by any of the teachers. Only Marie included expansion and factorizing of algebraic expressions in her lesson content.

## 6.2 Case 1: John

John is a middle school teacher who began his career as a primary teacher. He has been teaching for 15 years and he has been teaching algebra at some level throughout his career. John's school is a large independent school in the northern suburbs of Perth, Western Australia. In 2012 the NAPLAN testing results in Numeracy showed that the number of students in Band 7 was statistically above the Australian average of Band 5 for Year Seven students. There is an assessment scale for each year level tested and a minimum standard is defined. Students are tested in Year Three, Year Five, Year Seven and Year Nine in Australian schools. The highest band a student can be awarded in Year Seven is Band 9. The school offers a wide range of courses and in recent years a significant number of students have chosen to enrol in university when they leave school.

Table 28 which follows provides the reader with an overview of the content John covered in each of his lessons. The information is taken from notes the researcher compiled.

Table 28 John's lessons-Researchers notes

<p><b>Lesson 1:</b> Find a suitable definition for algebra Emphasis on positive feeling about algebra. Links made to primary number. Magic trick find the missing number questions. Discuss the notion of variable for the unknown number. Solving equations using inverse operations is modelled. Emphasis on balancing both sides of an equation. Exercise set from textbook (using the calculator with negative integers discussed students will do more work on this later in the year). Magic square projected on the IWB. Can students remember this pattern? Toothpick pattern projected on IWB. Problem demonstrated by the teacher using toothpicks on tables at the front of the class Students asked to look for a relationship and to create a formula that can be used to predict number of toothpicks. Students work on questions from textbook. May use toothpicks if they wish.</p>	<p><b>Lesson 2:</b> Number patterns on IWB. Students asked to Continue the patterns. Do they recognize any of them? Answers given. Recap of the week's lessons in algebra. New pattern introduced using shapes. The shapes are polygons and the students need to find the relationship between the number of sides of the polygon and the number of triangles found in each polygon. From this the sum of the angles can be found. A challenge is issued. Invent your own rule. Students have to give a list of numbers in their pattern and the teacher has to guess their rule. Students are given a Blank T shirt page and some cardboard. The students then work on creating tables and rules to continue a pattern. Students are told that they will develop their work on rules and that rules can be used to find answers.</p>
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<p><b>Lesson 3:</b> Each student is given a small bundle of cubes. The students will be using their graph books and notebooks this lesson. The teacher uses the painted bricks problem with the class. How many faces are painted? Students have to create a rule and a table of values with the teacher's help. Teacher models how to draw a linear graph. Instructions are given about scale, lengths and labelling the axis x and y. The Cartesian plane is drawn with no mention of its name. Students plotted points on the graph but did not use any of the conventional terms to describe what they were doing. They were instructed to join the points to draw the line and the teacher demonstrated how this line could be used to predict the value of y given the value of x. The class appeared to have no difficulty in doing this. Modelled using an excel spreadsheet to generate different values of x. Students put a formula into an excel spreadsheet. Students practiced from textbook questions using excel spreadsheets.</p>	<p><b>Lesson 4:</b> Students have to save all maths worksheets from this unit to their e drives on their laptops in a folder called perhaps patterns and rules. Eric the sheep task from maths 300 used. Students acted out being the sheep and one student was ERIC. After the class had gone through an enactment of the problem they went to their desks and began to use cubes to re-enact the problem. Students needed to have their books open and to write things down. They had to work out the number of "goes" it would take ERIC to get to the front of the shearing line. Trying to find the pattern was a class effort with help from the teacher. It proved to be a challenging task. For the remainder of the lesson the students took a short diagnostic test from the textbook.</p>
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### 6.2.1 John's beliefs and practice for students' development of number concepts

John believes that good number skills are the basis which allows Year Seven students to achieve success in algebra. His experience in teaching in the earlier years of schooling has impressed on him how this can be achieved with even the youngest students.

*I think mental maths should have a stronger emphasis. So when you come to do things like this and you are looking at a pattern 2, 4, 6 whatever is the pattern you can quickly work out the difference or you can quickly do 2 steps times by 2 and add one whatever the pattern is. (Interview 1, March 2012)*

Flexibility in dealing with numbers, that is, being able to calculate, order and compare, is seen by John as being important and he has used a variety of methods to assist students in developing the necessary fluency. John sees no need to rush to pen and paper activities to calculate using the four number operations, but rather he advises that skills can be fostered by using both manipulatives and games. Regular practice is seen as crucial and when the activity is seen as fun students are more likely to engage and succeed.

John views algebra as important and he enjoys teaching it. It is his view that pattern recognition and being able to continue patterns are key ideas that can begin to be developed in the early years of schooling. Good number skills are also seen as being an important precursor to algebra. John wants his students to be able to use number effectively and he uses a variety of practical activities in his teaching to help students to develop their understanding of number and their ability to mentally manipulate whole numbers. John has taken this belief of the importance of number to the beginnings of algebra to his work with Year Seven. John consults with the head of mathematics at the school on a regular basis and is therefore fully aware of the expectations Year Eight teachers will have about what content has been covered in Year Seven. John is also mindful that some students who come into Year Seven will not have the prerequisite number skills to assist them in beginning algebra. John talked about the fact that in his school, time available for remediation of number skills in the week is limited by the timetable. He sees his students for mathematics lessons four times a week. The lesson duration is 55 minutes. John teaches a class of 32 Year Seven students.

*Quite often their basic number skills actually are there, their mental maths capacity isn't there. I don't think we spend again enough time teaching those mental strategies early on. The ability to look for patterns, the ability to count on ... to count backwards, the ability to manipulate numbers. We don't do enough of that at times. (Interview 1, March 2012).*

With the adoption of a national curriculum Australian schools may well struggle to meet demands in terms of expected standards. The Australian Curriculum has been planned on a yearly basis. What happens when a Year Seven student is not ready to move to the Year Seven curriculum? John raised the issue of lack of time to do some remediation with students, who had not mastered number skills by the time they entered Year Seven, at the focus group meeting.

*There is this issue that you don't have the time to do the remedial work. That you, at times, need to. There are students who come up with a diverse range of abilities. I know that we find that very hard to support and go over some of the basics. (Focus group meeting, November 2012)*

John also strengthened his position on the importance of good mental arithmetic skills and understandings when the four teachers in the study raised the question of the use of a



calculator in class in discussion.

*They still have to know what they actually are doing to put it into the calculator They still have to have that fundamental knowledge of the basic concepts. (Focus group meeting, November 2012)*

Figure 21 shows the two themes of practice John used when teaching number concepts.

John's themes of practice for teaching number were seen to be Student Focus and Teacher Focus.

TEACHER FOCUS		
Code	Strategy	Orientation
23	Use of practice in class	Transmission
36	Direct instructions	Transmission
17	Definitions presented and explained	Transmission
15	Whole class instruction	Transmission
STUDENT FOCUS		
Code	Strategy	Orientation
42	Allows student to investigate	Discovery
18	Students make their own notes	Discovery
35	Personalized Learning	Discovery
24	Follows textbook, worksheets	Discovery
25	Students work individually	Discovery

*Figure 21 Teacher focus/student focus: John's themes of practice in teaching number*

In Lesson 2 John taught the class as a whole. The task students were given—devising their own rule and then generating numbers—gave the students the opportunity to practise what they had learned about patterns and rules.

*Your task is to go away and think of a rule a pattern rule that you think I won't be able to get or other people. You have then got to apply it to some numbers and we are going to put them on the t shirts. (Transcript of Lesson 2, August 7, 2012)*

This was essentially a task for individuals but the students worked at tables and this resulted in students discussing their work as they struggled to find the best pattern for them. For some students this required much trial and error as they endeavoured to make their pattern hard for others to find the rule.

*You can make it as tricky as you want to try and fool other people. You might want to work out on a piece of paper first what your rule is and just check it out. (Transcript of Lesson 2, August 7, 2012)*

When students had completed their T-shirts they hung them on a line.



Figure 22 Student work

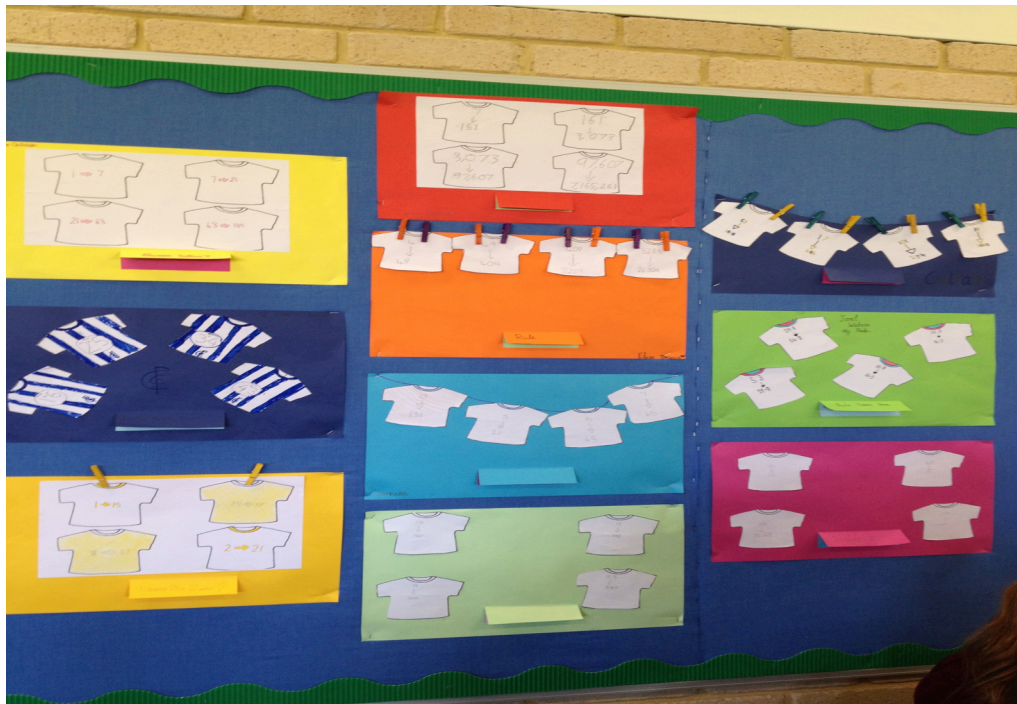


Figure 23 Students' work

The students were particularly engaged in creating their number washing line. The Researcher observed purposeful activity and they appeared to enjoy the freedom of discovering their own rules. One student discovered that there was not only a pattern to be found in the numbers, but also in the design of the washing lines. John wanted to develop this idea of pattern a little further and he projected a table on the interactive whiteboard, which involved polygons and angles. This information was familiar to students because of earlier work they had completed on shape. John explicitly made links with number work in primary school, as the students were all familiar with the "find the missing number" question where the missing number was usually denoted by a square. John also reminded the students of the 'working backwards' strategy they had used earlier in the year.

*Absolutely you do have to work backwards and that is one of the problem solving strategies we have mentioned this year. Definitely in algebra you have to work backwards in some of the examples we'll have today. Even this example, you need to work backwards in order to solve it. Bigger numbers, it can get a bit confusing. (Transcript of Lesson 1, August 3, 2012)*

John then set the students an exercise on solving equations using inverse operations from the textbook. The students completed their own solutions but they were able to work with others if they chose. However, students essentially worked alone, consulting a fellow student from time to time. The structure of the program of learning used by John for his students indicates that whilst it was linear in design there was opportunity within the program for the flow of non linear enquiry. This supports John's beliefs about the nature of teaching.

### **6.2.2 Summary of John's beliefs and practice for students' development of number concepts**

- John believes that a good understanding of number and fluency in using number in the four operations is important.
- He provides his students with opportunities to practice their number skills both mentally and with pen and paper.
- John is careful to explicitly link current number work with students' previous experience.
- John believes the calculator should be used judiciously and that understanding should always be a precursor to its use.
- In teaching for students' development of number concepts the cluster of strategies most often employed by John were from the themes of practice of Teacher Focus (transmission strategies) and Student Focus (discovery strategies). It is notable that both of these clusters of strategies do not contain any from the connectionist orientation.

### **6.2.3 John's beliefs and practice about students' development of early algebraic concepts**

John believes that algebra is essentially about pattern. Students need to have as much exposure to pattern as possible. They need to look for pattern and then be able to continue it. The times table is one of the first number patterns that students learn. John sees using manipulatives as important for this stage. In moving from the concrete to the abstract, sufficient experience in using the concrete must occur before real progress can occur, according to Piaget (1952) and Vygotsky (1978).

*Obviously we do units of work where we teach pre-algebra. We are looking at patterns. We are looking at representing symbols and things like that so in my experience we teach this every year. (Interview 1, March 2012)*

In each of the four lessons that the Researcher observed, John focussed primarily on this search for pattern and the need to describe the pattern first in words and then in symbols. John was clearly purposeful in his lesson design to help students achieve lesson objectives.

*We do with the pre-algebra a lot of focus on the patterns and sort of working out what is coming next and that sort of thing. Then obviously we develop it into some sort of graphing. It is very much an introduction and obviously talking with the head of mathematics we are conscious of setting them up for Year 8 when they work with the calculators and they do an awful lot more on that. So we want to make sure they have a grounding and understanding of what algebra is. (Interview 1, March 2012)*

John was the first of the four teachers whom the Researcher observed in Term Three. The content John covered in this term did not deal with algebraic notation or conventions as a focus. These topics were dealt with as they arose within the problem being examined. John advised the Researcher that the students had previously explored much of the language of algebra used in the lesson, and that they were comfortable with the words used and their meanings prior to the first unit of work on algebra.

John sees that language is an important part of any learning in mathematics, and particularly in algebra. His students have their own mathematics dictionaries and they add to these as the need arises throughout the year. This is a practice that John has adopted regardless of the year group he is teaching.

*Any time I do a new maths unit the words that are going to crop up will always go in their spelling list. We do quite a lot in the spelling program to draw on the language. In my previous studies that I have done in the past and presentations I've done [I've] talked about language. We should almost teach mathematics like a foreign language sometimes students only come to the usage of the word when it has a meaning for them.*

*The learning comes from the language they need to have that language hence why they probably were okay with some of those terms because they knew what the words were. (Interview 2, November 2012)*

When asked if there were any particular hurdles in the study of algebra which he had observed students experience in the early work, John replied:

*Building the stepping-stones for them sometimes you have to go back to the early stages for them to realize, Ah! (Interview 1, March 2012)*

John also believes that using a variety of strategies to teach early algebraic concepts is helpful to students. By taking this approach students with different learning styles are catered for and a certain depth of understanding can be achieved when students look at problems and ideas from different perspectives. It was evident in the lessons that the Researcher observed that this was a strategic decision John had taken.

In teaching for students' development of algebraic concepts the cluster of strategies most often employed by John were from the theme of practice of Teaching/learning balance. This strategic approach employs strategies, which arise from all three of the orientations of transmission, discovery and connectionist.

Table 29 *Teaching/learning balance: John's theme of practice in teaching algebra*

TEACHING/LEARNING BALANCE

Code	Strategy	Orientation
34	Allows students to make errors	Discovery
47	Students work with others	Discovery
37	Closed questions	Transmission
40	Teacher assessment	Transmission
27	Connections made between ideas	Connectionist

The lesson John chose as being one of his most successful was the third lesson observed by the Researcher. In this lesson John took the students from the concrete activity of creating a pattern using 2 cm cubes to finding a rule, and then describing the rule using a

graph. At the beginning of this lesson John placed a bundle of small cubes on each table. Each of the students was asked to take their own small pile of cubes to work with in the lesson and to recreate the sketch from the board.

*Is there anyone who can spot any sort of a pattern? There is no right or wrong answer. There could be lots of answers. Possibly.*

*How many faces or sides do I have to paint here? Clearly we don't have enough blocks to do that. But there is some sort of a rule there. If you have got it just put your hand up and I will come and look over your shoulder. It has to work for every single one of the patterns. (Transcript of Lesson 3, August 10, 2012)*

John then wanted students to graph their results on the number plane. The students had learned how to construct graphs earlier in the year. John reminded them of this work.

*Last term we did quite a bit of work on graphing. In what context did we do the graphing? What were we doing that for? We were doing it for the weather and what type of graphs did we do?*

*Would someone who has worked it out like to explain to the others what the rule is and how the rule works? Now the trick was, always remember, does it work for all of them? (Transcript of Lesson 3, August 10, 2012)*

Until this lesson, the patterns the students had considered were either concerning only numbers or with concrete materials. The aim of the construction of the graph was to assist students to see that a relationship could be represented in another way on the Cartesian plane. This lesson had its foundation in the previous lessons. Students were successful in making the transition from concrete to abstract and in generalising their findings to predict how many walls would be painted for  $x$  amount of cubes. John used technology with the students to assess students' understanding of the rules created, and as another medium in which students could describe their pattern.

When asked to reflect on his most successful lesson in his opinion John replied:

*The session where we started with the 2 cm wooden cubes, these cubes were used to create a pattern that the students then had to work out. We then*

*progressed, once we had found the rule, to graphing the rule. Finally we progressed to using an Excel spreadsheet to calculate future numbers for the students. (Reflection Question 1)*

The reason given by John when asked what he thought made the difference was:

*[t]he hands-on nature of the activity and then the variety of learning methods in it. This helped to cater for a range of students and their different learning styles, it also helped to consolidate understanding of one concept via different means. (Reflection Question 2)*

John was pleased with the progress many of his students made in the unit of work. If there were any concerns the major one was a lack of time to consolidate learning.

*There are always things that would, could be done differently, often that is as a result of constraints of time, as ideally I would spend longer on this unit, to really consolidate the understanding. A reassuring aspect of doing these lessons was the end of unit test. The results were highly positive, a class average of 73%, with a range of 20% to 96%. (Interview 2, November 12th)*

The Researcher, in the final interview with John, raised the question of students' progress. This interview took place in late November 2012 and came after the focus group meeting. It was obvious from the responses made by John in the second interview that he had reflected both on the discussion of the focus group meeting and the ideas he had been presented with in the Powerpoint presentation on teacher beliefs and practices.

*But [I] went away afterwards thinking it was amazing how many different ones there are. Sometimes you just do automatically and probably don't I know I don't analyze what I am doing, you just do. It's ingrained; you've done it for x number of years as you said. So, I was quite surprised that there was 20 to 30 items on the list there. I would reference it when you're justifying it to reference it to an example if I'm talking and here is the example of the close question which shows it is discovery transmission discovery. As a tool to teachers we all said we would like a copy at the end. It would be very useful to*



*manually go through. You have to get a balance. I have found myself analyzing myself in the last couple of days, what I am doing. (Interview 2, November 2012).*

John sees mathematics as a body of knowledge, which is built on each year by the students. However, time constraints can make it difficult for the teacher to devote enough time to each unit of work.

*We have got four lessons a week. It's not long enough in my opinion, we need one lesson a day. (Interview 2, November 2012)*

This was an issue that more than one teacher in the study raised and it will be examined in more detail in the discussion chapter of the thesis. In the final lesson observed by the Researcher John asked students to investigate the movements of Eric the Sheep in the Maths 300 task. He wanted them to look at the way Eric was behaving and to describe Eric's strategy using algebra. They were to generalize.

In posing the problem of Eric the Sheep from the Maths 300 bank of resources to the students John worked with the whole class. John asked for volunteers from the class to act out the problem at the front of the classroom. John then asked students to try to model the problem for themselves using the bricks and counters on the table in front of them. The counter represented Eric and the bricks the other sheep.

*Even numbers and odd numbers maybe that influences it as well. So some are consecutive numbers. What I would like you to do, I have brought a few little toys for you to play with. I don't have lots of toy sheep I am afraid, but I only have little blocks. They will have to represent the sheep. So give yourself a few in front of you so you have a few sheep there. (Transcript of Lesson 4, August 20, 2012)*

John found that a variety of answers came from the students in response to the question.

*Can you then work out how many goes it will take Eric to get to the front of the line and then come and write your number on the board? The question for you is what is this pattern? How are we generating these numbers? Has anyone got any theories? (Transcript of Lesson 4, August 20, 2012)*

At the end of the lesson John gave the students a short end of unit review test. The students could use counters to help them.

John believes that learning is also an interpersonal activity in which students are challenged and that they arrive at understanding through discussion. John encouraged students to write about their thinking as they acted out the problem using manipulatives. The class results for this unit test confirmed for John that using a bank of different strategies had made a difference.

*We did a unit test at the end and the scores were actually very good for that which is reassuring, obviously talking about different strategies transmission and all those sort of things and it is good to get that feedback at the end to sort of confirm hopefully what we did had a positive because their class averages about 70%. So that was reassuring [that] it meet the outcomes. Yes it, I suspect, it was meeting outcomes and hopefully so as the different strategies the different lessons that were there as you saw me try to do, different activities not just transmission. (Interview 2, November 2012)*

#### **6.2.4 Summary of John's beliefs and practice about students' development of early algebraic concepts**

- John believes that pattern is at the root of developing an understanding of algebra. If the student can see the pattern they will be able to deal with algebraic concepts.
- John's lessons in early algebra observed by the Researcher supported this belief. This approach also was in tandem with the textbook, which John worked from in four lessons observed.
- John believes that using a variety of strategies to teach early algebraic concepts is helpful to students.

- John believes that students need to understand and become familiar with the language of algebra.
- John plans lessons where students have the opportunity to move from working with concrete materials to abstract ideas.
- John believes that the use of technology can assist student understanding and engagement with early algebraic concepts.
- In teaching for students' development of algebraic concepts the cluster of strategies most often employed by John were from the theme of practice of Teaching/learning balance. This strategic approach employs strategies which arise from the three orientations of transmission, discovery and connectionist.

John planned for activities which allowed the students to trial ideas and to make errors. The students were able to work with others when they worked on a problem and the final summary was a collective effort. In guiding students' thinking, the questions John posed whilst seeming to be open were really closed questions. However, in giving students the opportunity to try a different answer he wanted them to come to the realization themselves what the answer should be. John explicitly made connections between ideas and assessed students' understanding using the responses he was given.

#### **6.2.5 John's knowledge of and beliefs about, the connection of established number (arithmetic) concepts with the effective teaching and learning of early algebra**

John is confident of his ability to teach for the transition from number to early algebra. He believes that his teaching background has provided him with a strong basis in numeracy and he sees this as transitioning to algebra almost seamlessly. He often linked what he was doing in the observed lesson to students' earlier experiences in the primary school mathematics class.

*Why use a letter? We talked earlier in primary school. We used hearts boxes. Why use a letter do you think? We are using this language. English and Maths are interrelated. It is easier to write  $x$  than put a box. I think it's just an ease factor there. (Transcript of Lesson 1, August 3, 2012)*

John's belief in algebra following on from number and at the same time being a part of number development was evident in several ways. He linked the process of finding of a missing number denoted by a box in a number sentence to algebraic thought.

*I started as a primary teacher so Year three was when I started. Algebra is in everything isn't it so I have always been teaching algebra at some stage. It is not necessarily labeled as algebra it's just been there, its incidental I suppose. (Interview 1, March 2012)*

John was presently in his fourth year of teaching Year Seven.

*You want them to succeed. You want them to realise, "Well I did this in Year One I may not have called it algebra but I did it then. Now what I am doing is slightly trickier but I can still do it if I apply the same logic I had in Year One. I can apply it right now. Building the stepping-stones for them sometimes you have to go back to the early stages for them to realize, Aha! earlier. (Interview 1, March, 2012)*

John believes that learning algebra should be achievable for all students and something not to be feared. John was quite different in his approach to the other teachers in the study and this may be due in part to the fact that he is primary trained and the other three teachers are secondary trained. He was not afraid to change activities during a lesson. It is notable that in a fifty-five minute lesson he could have his students working on three different activities consecutively. Students easily moved from one to the other.

*I feel perfectly at ease with teaching algebra. It is not something that worries me. As you know you teach things quite a few times x number of years of teaching, you end up repeating certain things. I feel fairly confident in that I also feel confident within my classroom management to have the matchsticks out. Controlling different learning styles. There are some children who like the paper and pencil style. There are some who need to have the concrete stuff there. I feel very confident in that sense I suppose. (Interview 2, November 2012)*

John values giving students the opportunity in early algebra lessons to use concrete material, to explore the idea of pattern.

*We can begin to use a formula to try and help us plot the number of sticks we would need without having to build them all. In the book chapter 6.01 it asks us to carry on with the same thing. If you want to use the matchsticks, that is fine or if you want to work without them. (Transcript of Lesson 1, August 3, 2012)*

In three of the lessons observed concrete examples were used. The strategic approach adopted by John for this series of lessons in early algebra belonged to the theme of practice teaching/learning/student balance.

Table 30 Teaching/learning/student balance: John's theme of practice in guiding students' transition from number to algebra

TEACHING/LEARNING/STUDENT BALANCE		
Code	Strategy	Orientation
10	Connection to prior learning	Connectionist
22	Models procedures and process	Transmission
14	Asks students for reasoning	Discovery
13	Engagement focus	Connectionist

John used matchsticks with the students in Lesson One. He modeled the construction of toothpick houses with the students standing around him and also showed students how to use a table to find the rule to continue the pattern on the whiteboard. At the same time he showed students how to construct the shapes and the table and he asked them questions to stimulate their thinking from the ebook (textbook) about continuing a pattern using a table of values. He encouraged the students to identify the relationship between the number of triangles completed and the number of matchsticks used.

*So we have to work out some sort of rule between the number of the house in the pattern and the number of toothpicks?*

*Does that help anyone think of a rule? How many triangles are in here? How many sides has this shape got? For each of these numbers here has anyone spotted any relationship? (Transcript of Lesson 1, August 3, 2012)*

He used this discussion with the class to ask the students to look for the pattern in the task he was about to set them.

*So we should have a bit of a rule for the shapes we looked at last term. I can take any shape you wanted. Could you get your calculators out for me please? Could you work out for me how many degrees a twelve-sided shape would have using this rule? (Transcript of Lesson 1, August 3, 2012)*

John modelled the procedure for completing the table of values and how to use this information to predict the sum of all of the angles in a twelve-sided figure.

In Lesson Three and Lesson Four the students used small cubes. Lesson Three began by John placing a bundle of 2 cm cubes in front of each student. The students were asked to have their graph books and notebooks ready. After giving students some time to “play around” with the bricks John brought the class together and asked the students the total number of faces that he would need to paint if he were to paint the wall of bricks.

*Over the last couple of weeks we have been looking at various patterns and we have found some rules. And there is a pattern in front of you what is it? Is there anyone who can spot any sort of a pattern? There is no right or wrong answer there could be lots of answers. (Transcript of Lesson 1, August 10, 2012)*

The class were seated in groups but essentially this was an individual task. The teacher then walked around the room helping individuals who required help. Students made their own notes about what they had learned.

From these concrete beginnings students correctly constructed rules and formulae, and graphed co-ordinates.

*Every time you are looking for a pattern it helps to structure your thoughts to have a table something like that. You could probably predict what is going to*

*be for the 4 and the 5. What did you have for 4? Have you spotted anything? What I would like you to do underneath here is have a rule for me. So if you can try and write a rule for me of what is happening? (Transcript of Lesson 3, August 10, 2012)*

In Lesson Three, where students graphed the pairs of numbers this idea was expressed as a natural progression from the work John had done earlier with the students on drawing up a table to answer a question. No mention was made of coordinates or even about the Cartesian plane, and yet the students were able to graph the points and predict the outcomes.

John set the class an exercise on using spreadsheets to continue patterns once the rule had been entered. In using technology to describe a pattern, John engaged the students and this mode of enquiry gave all students the opportunity to participate fully in the exercise. Once again he modelled the procedure, while at the same time asking students questions that required them to think about what they were doing. He guided student thinking to make the connection between earlier concepts and the present.

*How are we going to do it on the computer? Somehow I have got to use this Excel which we have touched on before remember we did a shopping thing before and put in a spreadsheet? Think about this rule and how we could put it on there. Have a column for x and a column for y. Now did you realize that I don't have to type all those numbers? It will do all those consecutive numbers for me.*

*Get your desk set Excel up and I will step you through what you need to do. This formula here, do it with me if you need to. (Transcript of Lesson 3, August 10, 2012)*

In modelling the process to solve an equation, John demonstrated that he also believes that mathematics is a body of knowledge and that its procedure and process, accepted by the wider community of mathematicians, needs to be modelled to students. John's professed beliefs about that mathematics was demonstrated also, in analysis of the quantitative data from this study in section 5.3.

In the fourth lesson observed by the Researcher John set the class the problem of 'Eric

the Sheep', which comes from the Maths 300 lessons. How many 'goes' would it take for Eric the sheep to get to the front of the queue? After enacting the problem with the students, John asked the class to search for a pattern. How many sheep were in front of Eric? How many places did he move? John was here using the strategy where he modelled procedures and processes. After some discussion the students were asked to return to their desks, and John then gave every student a small pile of cubes and a counter. John advised students to have their own books open and to write notes down. Each student could now enact the problem with the manipulatives in front of them and record their findings.

Trying to find the pattern was a class effort with help from the teacher, who used advice and questioning, as well as telling students that pattern could be found.

*Anyone else got a theory? I see where you are going..... it seemed to break down a little bit then. Anyone got a theory? I want to try and get answers. Couple of people have said [to] me 4 is a magic number. So there is something to do with 2 sheep. And you are sort of saying plus or minus here. Not sure how we use that. I would say that is good. Have a go at that one. The number of sheep plus 2 divided by 4. See if that works for any of those numbers. Pick a number and see if it works. Why doesn't it work? Because there is a remainder. I will put in a little bit of a rule on this one. We have to round up we can't deal with parts of sheep. So by talking about it we generally got at most of the information we needed. Quite a hard rule to put together. (Transcript of Lesson 4, August 20, 2012)*

This lesson was timed at the end of this first block of lessons on early algebra. The class had worked with constructing rules and making predictions. The class was engaged and the variety of responses indicated they were engaged with some depth of thought. In using this cluster of strategies John demonstrated his professed belief that understanding is important to student learning, and whilst the teachers' role involves transmitting knowledge it is important for the teacher to ensure that the students connect with the content in a meaningful way. In using this cluster of strategies, John demonstrated his belief that a balance of strategies arising from the paradigms of both transmission and constructivist are what good teaching is about. Further, in using this particular cluster of strategies he also included those from an orientation of connectionism. The quantitative data gathered from John shows that he



sees understanding as important, and making connections between ideas as important. In the final interview, which occurred shortly after the focus group meeting, John had begun to reflect on his lessons in the context of the cluster of strategies he used and the paradigms from which they originated.

*You have to get a balance. I have found myself analyzing myself in the last couple of days, what I am doing. (Interview 1, November 2012)*

John values watching others teach and he is willing to invite teachers into his classroom to observe his teaching practice. He sees this as an opportunity for professional development as a teacher. John said that:

*When you go in and watch other people and you can see how they interpret the same sort of concepts we sort of deliver in different ways. We don't have enough opportunities in our profession. I don't know what you do in your schools. You can see how others approach different units of works in our own styles. (Focus group meeting, November 2012)*

#### **6.2.6 Summary of John's knowledge of and beliefs about, the connection of established number (arithmetic) concepts with the effective teaching and learning of early algebra.**

- John's willingness to be involved in a study that explores teaching strategies for the transition from number to algebra supports his commitment to continued learning.
- He is familiar with the Maths 300 material (Education Services Australia, 2010), which provides teachers with engaging and open-ended lesson ideas. John is appreciative of the teachers' notes provided.
- John has made presentations to other teachers and he is confident about sharing his ideas with others.
- John believes that learning algebraic concepts is achievable for all students and in the lessons observed by the Researcher John taught the class as a whole.
- John is confident about issuing materials to students to allow them to work with ideas in a creative and constructive way.

- He ensures that students are engaged in the lessons and asking questions was encouraged.
- John makes transitions within each lesson for students to move from one task to another with the minimum of disruption, which demonstrates his organizational skills and his wish to engage students.
- John believes that mathematics is a body of knowledge and that procedure and process accepted by the wider community of mathematicians needs to be modelled to students.
- The strategic approach adopted by John for this series of lessons in early algebra belonged to the theme of practice of Teaching/learning/student balance.

John employed strategies from all three of the orientations identified by Askew et. al., (1997). John worked with the class as a large group in each lesson, but also provided opportunities for students to work individually or in smaller groups. The students were grouped around tables so this facilitated flexibility of approach. John used concrete materials to foster student growth in understanding the role that algebra can play in describing the physical relationships they had found. In his use of technology John also provided the students with a pictorial representation of the relationships they had found. In using spreadsheets John modelled how these could be used to generate possible values and to make predictions, showing the students the power and possibility of algebra.

Table 31 shows data gathered from the four lessons observed by the Researcher of the previously defined strategies. The percentage weighting of each in each lesson is also shown in the table in the columns headed T, C and D.

Table 31 Overview of John's practice

Lesson	Transmission	Connectionist	Discovery	% T	% C	% D
1	9	3	6	50	17	33
2	9	5	7	40	25	35
3	8	3	8	42	16	42
4	5	3	6	36	22	42

The features that bind these lessons together and identify them as a balanced lesson are

(i) the use of connection to prior learning; (ii) asking students for reasoning; (iii) maintaining an engagement focus and, (iv) connecting the teacher and student to the learning episode in a meaningful way. In a balanced lesson the teacher and student have almost equal input into the teaching/learning that is happening in the classroom and the teacher ensures that this balance of teacher focus (transmission) strategies and student focus (discovery) strategies is brought together by the skillful use of connectionist strategies. This helps to make the learning memorable for the student. The connection occurs because the continuing dialogue between the teacher and the students throughout the lesson encourages them to connect their thinking on two levels. The student firstly is asked to recall previous knowledge, understandings and applications, and then secondly to be open to building new knowledge and understanding and to apply this new learning. The use of the strategies of modelling procedures and processes teaches the students the correct algebraic notation to describe their thinking.

### 6.3 CASE 2: GRACE

Grace is a secondary teacher of mathematics who is also Head of the Mathematics Department in a large independent school in Western Australia. The school has students from a wide range of ethnic backgrounds and caters for students for Kindergarten to Year 12. The school has comprehensive educational and co-curricular programmes. In 2012 the NAPLAN testing results in Numeracy showed that the number of students in Band 7 was statistically above the Australian average. Grace has taught mathematics for approximately 14 years in total. Initially she taught for three years, then she moved to other employment and returned to teaching in 2000. The Year Seven class of 21 students Grace worked with in this study had been identified as an able group of students and their mathematics lessons were for 75 minutes duration three times a week. Three of the four lessons observed by the Researcher were held in the same classroom. As Head of Mathematics Grace is responsible for mathematics from Kindergarten to Year Twelve at the school. Grace and her teachers had been preparing for the mandate of the Australian Curriculum and had been using this document in their planning for several years.

Table 32 which follows provides the reader with an overview of the content Grace covered in each of her lessons. The information is taken from notes the researcher compiled.

Table 32 Grace's lessons-Researchers notes

<p><b>Lesson 1</b></p> <p>Start with a dice game. The object was to make 2 three digit numbers and then find differences. Work on definitions of words written on the whiteboard. Constant, variable, co-efficient, term, expressions and equation. Discussion of the meaning of each word. Examples were also given and discussed. Students made notes. Students take a 10 question quiz on the terms they have defined. True or False.</p> <p>The quiz is marked. Questions taken from the textbook are written on the whiteboard. Students have to identify terms, co-efficients, variables, constants, equation or expression. It was pointed out that negative numbers were covered at the beginning of the year and at least one student needed to revise this topic.</p> <p>Finally students attempt another problem from the board. Translate a word problem to an expression then an equation. Substitution of values to find unknown.</p>	<p><b>Lesson 2</b></p> <p>Remember calculating perimeters and areas using formulae. Students came out to the board to write their favourite formula. Links made to familiar formula, recipes.</p> <p>Students to find perimeter and area of the square drawn on the board. Students need to write a set of rules before they calculate. Rules of order mentioned. Example of calculations worked by teacher. Students calculate the area etc of their shape.</p> <p>Next phase of lesson students' work on using rules, creating tables and graphing straight lines. Teacher models several examples on the board that students also work through. Students are given a worded problem and Asked to write an equation. Solutions are marked. Teacher goes through with the class. The same procedure is used with two more questions from the textbook.</p>
<p><b>Lesson 3</b></p> <p>Activity using dice-activity involved substitution into a formula. This activity follows on from the previous lesson. The body of the lesson will be about rules. Replacing variables to find the solution to a problem. The procedure is modelled and students are questioned to check for understanding. Students then work on examples from the ebook. Order of operations mentioned in response to a question. By the end of the lesson students are writing equations and using them to solve problems. They work through specific questions from the text and these are marked in class together.</p> <p>Introduced to flowcharts. Then students practice constructing flowcharts to create equations by doing questions from the textbook.</p>	<p><b>Lesson 4</b></p> <p>Focus of this lesson was patterns. Working from the textbook, students are encouraged to find the rule to describe the pattern. An example is modelled – constructing triangles from matchsticks. Students copy example as the class is taken through the steps. One more question is done this way making sure everyone knows what to do. Then the class is spilt into groups. Each group is given a question from the text and asked to write and equation and then to use it to solve the problem. Each group explained their solutions in turn by writing up their work on the whiteboard. The remainder of the lesson the students worked through selected questions. The class stopped work when questions were marked together.</p>

### 6.3.1 Grace's beliefs and practice for student development of number concepts

*It is the next step up from your number skills when get your number skills in place then you can start generalizing into algebra. (Interview 1, August 2012)*

Grace also, like John, sees the importance for students to understand number, and the need for them to have gained sufficient fluency in using number before they can begin to generalise using algebra. When asked if she thought whether good number skills would guarantee success in algebra, Grace did not believe that this would necessarily be the case. To support her assertion, Grace cited her experience of teaching students that had worked well with number, but who had later forgotten their number skills, defined here as operations with number, when presented with the abstraction of algebra.

*I have seen it in good classes where they have really really sound number skills but suddenly we venture into the abstract area they forget their number skills. They probably do it less than kids who are not good with their arithmetic skills but you still get the same things coming through. (Interview 1, August 2012)*

In working with students who do not have good number skills, Grace believes that regular revision is important. In her position as Head of Mathematics at the school Grace has advised her teachers to include more practice of arithmetic skills in Years Seven, Eight and Nine classes.

*I think too it is something I am trying to address at the moment with arithmetic skills as well is that you do a little block of it and then you don't do it for a while. I have [been] trying to get staff to do a lot of immersion with arithmetic skills in Seven, Eight and Nine. And I think we will probably then lead that into going back and revising our algebra skills as well. (Interview 1, August 2012)*

Grace's theme of practice for teaching number was seen to be Teaching/learning balance.

Table 33 *Teaching/learning balance: Grace's theme of practice in teaching number.*

TEACHING/LEARNING BALANCE		
Code	Strategy	Orientation
34	Allows students to make errors	Discovery
47	Students work with others	Discovery
37	Closed questions	Transmission
40	Teacher assessment	Transmission
27	Connections made between ideas	Connectionist

In the first observed lesson Grace began the lesson with a dice game that gave students the opportunity to investigate three digit numbers using an eight or ten-sided dice. They had to find out how many three-digit numbers they could create that would meet the requirements of the task. The game was both personalized and inclusive as all the students in the class could play it. Closed questions assisted Grace in assessing whether or not students were on the right track with the game. As the game progressed Grace asked questions of the students, which prompted them to think more deeply about what they were doing in terms of finding the biggest difference or the smallest difference. Students were able to correct their own errors as they played the game. The game allowed students to think about the construction of three-digit numbers and how place changed digits' values as they were moved.

In the third lesson observed by the Researcher Grace introduced formulas that had a practical use. The numbers involved in the calculations were decimal and squared numbers and this called for calculator use at this time.

*You should be doing body mass index with this formula. This is an actual formula that they do use to calculate body mass index. And then we have got our speed formula and this is the actual formula you use to calculate speed. If you haven't met these already in Science and Phys Ed you are going to in the next couple of years. (Transcript of Lesson 3, October 30, 2012)*

Grace was committed to making improvements in number skills and number sense of the students in her school and this was supported by her contribution to the group discussion on the subject of calculator use at the focus group meeting, held towards to the end of the study. Another teacher at the meeting suggested that, if poor mental arithmetic skills were

preventing a student from dealing with early algebra, then a calculator could be used. Grace did not share this view and stated that to neglect the practice of mental arithmetic skills would not serve students well in the future years of schooling, particularly if they intended to study mathematics at Stages Two or Three in Years 11 and 12. In Western Australia the examination papers for courses at the Stage Two and Stage Three have a calculator free section of the examination paper.

*I think we still persevere because we know the end point is calculator free exams. We do, do remediation across Seven to Ten and yes some of their assessments are modified yet where possible we are trying to improve their mental maths skills. (Focus group meeting, November 2012)*

During the final interview and in the reflective questions submission at the conclusion of teaching the unit on beginning algebra, Grace reflected on the need to review her number work with students before she began to teach such a unit of work in the future.

*One thing I would probably do differently before the start of the unit that I was working with is build in number. I didn't do it and the kids didn't suffer because they are fairly bright and retentive anyway. I probably would have built in some revision of order of operations square numbers and that sort of stuff. And negative numbers. (Interview 2, November 2012)*

*I was able to remind them of things on the fly. But if it was a different level class then I would probably make it more formal ... have some mental maths beforehand. Have one lesson we will do all those things beforehand or have some sort of game that to play that reminded them of those things. (Reflection questions response, November 2012)*

### **6.3.2 Summary of Grace's beliefs and practice for student development of number concepts**

- Grace sees that the development of number concepts is important to students learning algebra. She believes that this knowledge is necessary if students are to be able to learn to generalize.



- Grace does not believe that good number skills will necessarily guarantee success in algebra. She cites her own experience when students who had good number skills seemed to forget them when using algebra.
- Grace believes that mental arithmetic is important and that this should be an ongoing area for practice.
- Grace believes that use of a calculator is called on from time to time but the calculator should not be used if the calculation can easily be done without it.
- Grace gave students the opportunity to work with numbers in rules that did not involve only whole numbers.
- Where possible she used numbers from real life examples that she found in the text book she used with this class.
- On reflection, Grace said prior to beginning to teach this unit of work on early algebra in the future that she would include revision of order of operations and square numbers in her planning.
- Grace used games to investigate place value and the substitution of numbers into equations.
- When tackling problems that called for substitution of decimals Grace suggested that students should use calculators.
- Grace encouraged students to identify any number patterns in the tables that they constructed.
- Throughout the lessons students had the opportunity to work on their own or with others.
- Grace used closed questions to assess students' understanding and asked questions to further student thinking about number.

Grace's theme of practice for teaching number was seen to be Teaching/learning balance.

### **6.3.3 Grace's beliefs and practice for student development of early algebraic concepts**

Grace sees the need to scaffold student learning in structuring early algebra content to suit the learning needs of the students she is teaching. She is concerned that students overcomplicate their work by combining too many procedures. It is suggested that when

students do this it is indicative of a poor understanding of process. Using algorithms without understanding the reasoning that underpins the procedures in numerical contexts may well lead to confusion and failure to make correct use of the algorithms when transitioning to algebra.

*I think they try and over complicate it the young students do. You teach one or two algorithms and suddenly they try and combine the whole lot together.  
(Interview 1, August 2012)*

*Generally I have found with Sevens and Eights. They are quite comfortable in medium of working with terms and variables. Unless you are quite specific in how you differentiate out what you do with addition and subtraction and what you do with multiplication and division they try and bundle it altogether.  
(Interview 1, August 2012)*

Grace feels that it is important to model the correct use of algebraic conventions so that students clearly understand the rules governing addition, and subtraction, and multiplication and division. The Australian Curriculum document states this clearly as a component of the Year Seven achievement standard:

Year 7 achievement standard

They connect the laws and properties for numbers to algebra. (ACARA, 2013)

In her position as the leader of mathematics in her school from K to 12 it is reasonable to accept that Grace has a good knowledge of the curriculum content in mathematics for all years within her school. Her teaching experience has provided her with the understanding and insight necessary to effectively tailor curriculum content to student needs.

The strategic approach adopted by Grace for this series of lessons in early algebra belonged to the two themes of practice, Teacher Focus and Teaching/learning/student balance.

TEACHER FOCUS		
Code	Strategy	Orientation
23	Use of practice in class	Transmission
36	Direct instructions	Transmission
17	Definitions presented and explained	Transmission
15	Whole class instruction	Transmission
TEACHING/LEARNING/STUDENT/BALANCE		
Code	Strategy	Orientation
10	Connection to prior learning	Connectionist
22	Models procedures and process	Transmission
14	Asks students for reasoning	Discovery
13	Engagement focus	Connectionist

Figure 24 *Teacher focus and Teaching/learning/student balance: Grace's themes of practice in teaching algebra*

In one lesson in the unit of work on early algebra Grace was not particularly happy with the outcomes of the lesson. On reflection she believed that she might have confused students with her introduction. Grace sees that it is important not to teach too much too soon, and to carefully pace delivery to meet students' needs.

*The one lesson I wasn't hundred percent happy with was the creation of rules from tables. But I think that was me not them I think I confused them at the beginning. I think I am more into, I am getting more and more ideas and how I can make it start from a place that they understand and move into an abstract so getting at the concrete ideas I think are better each year. I do this and the games and the things that make it interesting for them. (Interview 2, November 2012)*

Grace believes that the use and development of several key concepts in early algebra can be discovered or derived by students when the need arises. She prefers to put in place scenarios where students can work through a problem in their own ways, which she believes would naturally lead them to using algebraic thinking.

*I think that these are great. I think we still have your open ended scenarios is*

*another way to actually go about teaching some of the algebra content. I am thinking specifically of things like the graphing, where they can actually collect some linear or create some open scenarios where they can actually...They would automatically Year Sevens would automatically get the information together they would probably put it in a table. They probably would graph it. These are things that they would automatically do. (Interview 2, November 2012)*

Grace has experience of the negative view of algebra that students can bring to the class even before they begin to work. When parents tell their children that they could never do algebra themselves at school, it often does not encourage a positive mindset to its study. Using games and investigations in her teaching are ways that Grace sees can motivate and engage students and help them develop a positive view of algebra and its uses. Grace argues that students need to be able to communicate algebraically and use the language of algebra correctly. In the first lesson observed by the Researcher, Grace introduced students to some key words used in algebra by instructing the whole class. She wrote a list on the whiteboard under the heading Algebra Words, and included the following words: Constant, Term, Coefficients, Expressions and Equations. Students copied the words and the subsequent definitions and examples into their notebook or onto their ipads. Grace explained the meanings to the students, referring to previous work on the language of algebra. She posted a card with an example of each alongside each word on the whiteboard. Grace then asked the students to provide her with any other examples, which they did.

*One of our letters was a variable that had the opportunity to take on different values. So we could start off by saying a constant could be mathematically a number by itself. The meaning doesn't change. For example, the number 7 is a constant. Next one we have is a term. Now I know there are lots of English meanings for the word term. We are in the middle of a school term, term of a contract. In algebra term has a very specific meaning. It is basically a combination of letters and or numbers. Yesterday the letters we discussed had a very specific term. What were those letters called? (Transcript of Lesson 1, October 24, 2012)*

Grace believes that pattern is something that Year Seven students would have been

working with throughout their primary schooling, and more generally Grace and her teachers have been working with early algebraic concepts for the past four or five years in Year Five, Six and Seven classes, according to her understanding.

In Lesson Four Grace worked with the class specifically on patterns.

*So today is going to be all about patterns. All about patterns, but it is going to be a bit different from what you have done in Years Five and Six where you get a fill in the blank pattern. Today we are going to look at some patterns that we create and then make tables. (Transcript of Lesson 4, October 31, 2012)*

Patterns using matchsticks and fence posts were then described using tables of values. The class was taken through the steps in thinking required to 'find the rule from the table', namely, finding the difference pattern so as to locate the coefficient of the variable, and then finding the constant. Students then worked on additional similar questions to practise what they had just learned. Grace marked the questions with the students as they completed them. Her prior experience has shown that it is in the writing of rules to describe the patterns where the major difficulties may arise. The Researcher has also found this to be an area of challenge in early algebra. Grace would like to do more work on this aspect of patterns in the future.

*I've always worked on the basis though that in primary school they do pattern work, they do missing number work. They have a sentence with a box and put the number in the box and that has always been my starting point. We've been working with algebra with our Year Sevens for four or five years in our programs now. I knew this was coming with the Australian Curriculum and we adjusted our programs accordingly, quite early on now, with back-to-back for Six and Five as well. (Focus group Meeting, November 2012)*

Grace believes that number and algebra concepts have been recognised as a major component of the Australian Mathematics Curriculum.

*My thought is really, when you start breaking up the content of the Australian Curriculum it is heavily weighted towards number and algebra as far as content goes, to the point that we are actually doing a numerical evaluation of*

*a year's cohort of work ... (the) emphasis is weighted on number in algebra. I don't have a problem with that. (Interview 2, November 2012)*

Grace sees planning lessons in early algebra in Year Seven for Term Four of the year as a preferred time for this work to be done. Her rationale is that students will already be familiar with the use of algebra incidentally, as several topics involve its use. Grace gave using formulae in measurement as an example.

*I think having it towards the end of the year is probably a good idea because they have had a lot of practical use. They have done their measurement so they have learned to substitute already and I took that as their foundation for substitution. The clever ones have rearranged equations to say well look if the area is this and it's a square, it must be and so they are using their square root numbers to solve that and so they are already rearranging equations in their head. All you are doing is formalizing a lot of what they are doing intuitively. (Interview 2, November, 2012)*

Grace explicitly linked the use of formulae in the sciences and in recipes to the work she was doing with the class on equations.

*You know lots of rules in mathematics and in science as well. Can anyone think of any rules you would use in Science? Yes the volume one. Rules are important because they give you a guideline. An example of everyday rule might be that you are going to make some biscuits some cookies so your recipe is a set of rules it is a set of instructions isn't it. [If] you only want to make a small amount you can divide it by two if you want to make a bigger amount you could times it by two or three. You play around with it. (Transcript of Lesson 2, October 29, 2012)*

Grace assessed student understanding by her use of questioning and by actively listening to student responses. Grace also marked the examples students had been working on as soon as they had completed them. She saw it as being important to monitor progress in this way in the hope that any misconceptions could be found in the answers the students gave

quickly before they became habitual.

To identify likely errors was not included in the theme of practice Teacher Focus in the study but it had nevertheless emerged as a strategy arose from a transmission orientation. In the first lesson observed by the Researcher, Grace used quiz questions to highlight areas of possible confusion when dealing with order in using variables. She asked the question, “Is  $sr$  the same as  $rs$ ?” She then provided an example on the whiteboard using 3 and 4 as the values of the variables  $r$  and  $s$ , and she reminded students the operations carried out in number were the same as those in algebra because the variable or pronumeral represents a number. She did not define or name the Commutative Law. In the final task for Lesson One, students were to put into practice what they had learned about variables, expressions and equations.

*For those of you who have got a book out you can go to page 260 and go to Question 6. You will need to write the question down regardless of whether you have a book or not. The first thing we want you to do is to define the variable. Choose a letter to represent this distance that they travel. Write an expression to describe the distance travelled on the second day. You need to write an expression for the distance travelled on the third day. What doesn't get done will be homework. (Transcript of Lesson 1, October 24, 2012)*

In the second lesson Grace wanted students to be able use rules to complete tables of values. She taught the class as a whole and began the lesson looking at rules that the students were familiar with from their previous work in measurement. Grace then introduced the idea that tables of values could be created from a given rule. She drew up such a table on the whiteboard and showed the class how this could be completed. The names for the variables used were  $x$  and  $y$  and the rules were written in the form  $y=mx+c$ . Grace worked through several examples with the class and then the students were asked to attempt Question Four from the textbook, which was projected onto the whiteboard.

*Question Four, can we please do Question Four? Question Four is giving word sentences and would like you to write algebra equations. So you start it off with  $y =$  and then whatever we have got there a statement. (Transcript of Lesson 2, October 29, 2012)*

When the students had had enough time to complete Question Four, Grace went

through the solution with them. She followed the same procedure with Questions Six and Eight.

In Lesson Three Grace told students what they were going to be doing at the beginning of the lesson and reminded them of the work done in the previous lesson. The aim was to take students' thinking about rules in algebra to the next stage.

*Yesterday we worked on how to write rules and how to create rules. Today we are going to look at rules and what happens when we get the actual number for our variable, our variable being our letter. (Transcript of Lesson 3, October 30, 2012*

Grace instructed students on the method to be used to substitute different values into the rules she had chosen.

*So, here are the steps that we are going to do. We are going to write down the rule we are given, we are going to replace the variable with the number that we are given and then we are going to perform the calculation to get the answer. So for example, if we get a statement like  $y = x + 6$  and we are told on this particular occasion  $x$  is equal (=) to 5. I have written down my rule and what the  $x$  variable going to be. I then replace my variable  $x$  with what it is which is number 5 in this case and I have  $5 + 6$  and now I do a straightforward addition and say that  $5 +$  is equal (=) to 11. Is that quite clear? (Transcript of Lesson 3, October 30, 2012)*

When Grace was satisfied that all students understood what they had to do, she set them some questions to practice what they had learned. Grace reminded students of the need to be aware of the order of operations rule when tackling the set questions. At the end of the allocated time for this task Grace marked each question with the students, going through the method each time.



#### **6.3.4 Summary of Grace's beliefs and practice for student development of early algebraic concepts**

- Grace acknowledges the need to scaffold student learning in structuring early algebra content to suit the students' learning needs.
- Grace is interested in simplifying calculations for students so that they have a clear understanding of the rules for different operations.
- Grace welcomes the weight of the number and algebra strand in relation to the other strands in the national curriculum.
- Grace cites the importance of modelling the correct use of algebraic conventions to students so that they understand.
- Grace believes that key concepts in early algebra can be discovered or derived by students when the need arises.

The strategic approach adopted by Grace for this series of lessons in early algebra belonged to the two themes of practice Teacher focus, and Teaching/learning/student balance.

Grace used games and investigations to engage students' interest in the lessons and to help them to see algebra in a positive light. The students were encouraged by Grace to work through examples and to seek help when required. Throughout the series of lessons observed by the Researcher Grace worked with the class as a whole. Students were encouraged to communicate their reasoning when the class was working through problems together. Grace modelled how to translate a number pattern into a rule and then how to predict further values without using a table. In each lesson Grace used student practice as an opportunity for them to consolidate their learning. Lessons were structured to follow the design of the textbook. Grace's preference was to mark students' work with them as they completed it throughout the lesson. Grace believes that key concepts in early algebra can be discovered or derived by students when the need arises.

### **6.3.5. Grace's knowledge of and beliefs about, the connection of established number (arithmetic) concepts with the effective teaching and learning of early algebra**

Grace has experience of teaching algebra from Year Seven up to and including the Year 12 Specialist Mathematics course in Western Australia. In particular, Grace has taught Year Seven for three years, although not concurrently. She enjoys teaching this year group but this is not always possible because of timetabling restrictions.

*I love teaching Seven's when I can I try to get into that year group. But I had a couple of years when it clashed with the specialist maths, they were on the same grid lines. (Interview 1, August 2012)*

Grace is positive about teaching Year Seven and views early algebra as an important part of the mathematics curriculum. When asked why she took this view, Grace spoke about laying the foundations for future learning in algebra, and that as a teacher she enjoyed working with students who are in the early stages of learning about this strand of mathematics.

*I have just come back to it. I think it is more important, it's the most fun thing you can do because it leads onto calculus. (Interview 1, August, 2012)*

Grace is confident in her own knowledge of content, curriculum and pedagogy, and in her ability to meet students' learning needs.

*I have taught the Seven's at least three out of those five years so I'm quite confident in how to actually deliver it. I know what the Year Seven course looks like as far as recognition of patterns and grouping and I know where the Eight's are. I know not to overstretch it. I know where to pick up from. (Interview 2, November 2012)*

This confidence was supported, Grace believed, by the results achieved by the students in the end of unit assessment test. When Grace was asked if she had found what she expected to find in terms of the outcomes of student learning of algebra at the end of the unit she stated that student achievement was greater than she had envisaged.

*Yes I did because my students ... In fact I think because they actually surprised my expectations. They are a bright bunch of kids but they have just done their assessment. If you like I can give you a couple of copies. I will just blank the names out. We can do that before you go. On the whole they were able to solve equations, identify patterns write up the rules. It's really good. I was really, really happy and I felt at the end of it they were very confident about what they were doing. (Interview 2, November 2012)*

Grace's class average for the test was 82%. This compares to the Year Seven cohort average of 73%. She believes that she continues to improve her expertise in teaching early algebraic concepts to Year Seven. Knowing how to structure the content of lessons is something that Grace sees as important in assisting students to achieve success.

*I am getting more and more ideas, and how I can make it start from a place that they understand and move into an abstract so getting at the concrete ideas. I think it is better each year I do this and the games and the things that make it interesting for them. (Interview 2, November 2012)*

Also in the final interview Grace explained how she would not change her lessons in this unit of work if she were to be given a mixed ability mathematics class of students to teach in the future. She believes that the structure of the lessons and the strategies she used would work equally well with students from almost any class. Grace did qualify this by saying that if she were to teach those students who had been identified as needing extra help in mathematics that she would make some minor adjustments to the lessons.

*I don't know if I would ever be given just a straight class of mixed Year Sevens where there are different levels. I think having the way I approached it would be fairly, I think that the kids across any level would actually work. Focus kids, I would probably make it a little different because it would be more, I would probably try and make up things like cards and have more manipulatives for them to play with so it would be pitched at different levels. But that basic premise of how I would go about teaching would be the same. (Interview 2, November 2012)*

The Researcher would propose from her own experience that allowing students to investigate and to discover for themselves in a lesson requires the teacher to be both confident and knowledgeable. Otherwise valuable opportunities for learning can be lost and students can become discouraged. Grace expressed a preference for using strategies that could be characterised as moving from a teacher orientation towards one of student discovery. This was evident in her response to questions by the Researcher and in her responses to the questions asked in the early Questionnaire I on beliefs.

*I think we still have your open ended scenarios is another way to actually go about teaching some of the algebra content. I am thinking specifically of things like the graphing, where they can actually collect some linear or create some open scenarios where they can actually... They would automatically, Year Sevens would automatically get the information together they would probably put it in a table they probably would graph it. These are things that they would automatically do. (Interview 2, November 2012)*

In the lessons observed by the Researcher, Grace gave more weight to the beliefs orientation that teaching is chiefly concerned with transmitting knowledge. This may be due to the nature of the content covered in the program. Grace sees that where students are able, that is, when they have the prerequisite skills and knowledge, they should be given opportunities to investigate and discover for themselves. Her view is that these students come ready and motivated to learn and it is up to the teacher to provide them with occasions where they *can* learn.

*I think it's really important with those kids to provide them with situations where they can. They are so enthusiastic. They are so motivated. They are not jaded at all and they don't actually really care about being the cool kids when you first get them. You can set them scenarios and where they will go and actually find out things for themselves. I think the open ended. That is more the discovery as you say. Allowing kids to investigate. (Interview 2, November 2012)*

In the quantitative data gathered for this study Grace indicated that she believes

strongly that mathematics is a creative subject in which the teacher should take a facilitating role, allowing students to create their own concepts and methods. In using a dice game to begin the first lesson observed by the Researcher, Grace focussed the students on an activity as soon as they entered the classroom. The task she set them was achievable by all to some degree or another and built on previous learning. When the game was finished Grace asked students if they had developed a particular strategy to help them, and, if so, their reasons for choosing it. This allowed students to reflect on their thinking and to verbalise their choices. In this first lesson Grace also introduced each of the 'algebra words' and followed her instruction with ten questions to answer which would allow the students to consolidate what they has just learned. When marking the questions with the students, Grace asked the students for reasoning and made explicit connections to prior learning throughout the process. One particular example was when the question asked relied on the understanding that  $rs$  was the same as  $sr$ . Grace then set the students a task which involved them first identifying whether they were looking at an expression or an equation. When this had been decided they had then to list all terms, variables and constants that made up the expression or equation. Grace modelled this procedure by completing the first question on the whiteboard.

*Start with the first question. Is this an expression or equation? I am not going to race through them but I am going to explain just for the first one.*

*What is the co-efficient of  $x$ ? 2, 4 is  $x$  squared the same as an  $x$  ?*

*So we are only after the variable that is the  $x$  and  $x$  squared is very different from an  $x$ . Listing the variables we know we have got an  $x$  what else do we have? A  $y$  what else do we have? The  $x$  squared is very different from the  $x$ . Constants? 7, it has a minus or subtract sign in front of it so we probably have to treat it as a negative 7 if we are taking it just in context. So you have always got to attach the + or minus sign that is in front. Next thing is to list all the terms  $2x$ ,  $3y$  and  $-7$  and  $4x$  squared. I need you now to do the same thing for  $q$  2, 3, 4. You may very quietly discuss it with the person next to you. (Transcript of Lesson 1, October 24, 2012)*

Through her use of questioning Grace appeared to maintain student engagement and to scaffold their thinking. When students had completed this task and the work had been marked Grace set the class a problem. She told the students to write a statement using the variable  $x$  and  $y$ . Let  $x$  represent the number of boys and  $y$  the number of girls. Grace then asked for a

volunteer to come and write their solution on the whiteboard.

The students wrote:

$$x \div 3 = y$$

Grace showed the class another way of writing the equation namely

$$x = 3y$$

and asked if anyone had written the equation this way.

In this short teaching example Grace modelled procedure and connected students to prior learning also ensuring engagement as every student had to write down what they thought was the answer. She gave students the opportunity to talk about their reasons for writing the equation in the way that they did; and, most importantly, reaffirmed for the students the fact that both equations were correct representations of the worded statement.

In Lesson Two, Grace asked students to recall how they had calculated the perimeter and area of shapes using a formula.

*Try to remember as many formulas you have for areas of shapes. You can also do perimeter as well, so perimeter and area of shapes. You have got 5 minutes on that. (Transcript of Lesson 2, October 29, 2012)*

x	1	2	3	4
y	5			

Figure 25 *Table of values*

Grace modelled finding the area and perimeter on the whiteboard. She expanded on this idea of using a formula or rule by asking students to look at other rules. She explained that these rules could be used to describe a pattern and to continue a pattern in order to find the *n*th term.

Grace sketched a table of values on the whiteboard and gave the students the rule

$$y = x + 4$$

Grace then showed students how to complete the table to find the  $y$  values, thus modelling the thinking required to complete the task. She modelled this process again to give students the opportunity to ask any questions that they might have about the process, and to help them see a pattern from the table that was connected to the rule. Grace took the opportunity to make an important teaching point here, an advance organizer, pointing out that the difference pattern had significance and that this would become more apparent in future lessons. This could only assist student reasoning and engagement, the Researcher would suggest.

*Hands up if you can see a pattern in our results? It goes up by 5 and we can link that to our rule because I have got 5 lots of my variable. I would like you to turn to page 264 in your textbooks. [Page] 264, now I want you to have a quick look at what you are going to do...*

*Tell me what do you notice about the  $x$  values? Are they in nice neat order? You have to be careful about that and they have got the rule written out in words. (Transcript of Lesson 2, October 29, 2012)*

Grace moved on to the next step in thinking about using rules in algebra to generalize. This involved taking a worded sentence, translating it into an algebraic equation, and using this to solve a problem.

*I start with  $x$  numbers of lollies and the first thing I am going to do is I have to give half to my little brother so I have to divide by 2 then I eat 4. What does eating 4 do to my equation? So my rule is going to be equal to? (Transcript of Lesson 2, October 29, 2012)*

Grace projected this problem on the board and then modelled working through the solution with the class. All students were invited to work through the problem with her and encouraged to ask questions. Grace then asked the class to try two more questions on their

own. The starter game observed in Lesson Three was again designed to engage students and to give students the opportunity to practise the ideas that they had been working on in the previous lesson concerning using rules. In this lesson Grace's aim was to teach the class how to substitute into a rule or equation. She modelled how to deal with an example on the whiteboard, at the same time talking to the students about her reasoning as she went through the procedure.

*So today, yesterday we worked on how to write rules and how to create rules. Today we are going to look at rules and what happens when we get the actual number for our variable, our variable being our letter. So, here are the steps that we are going to do. We are going to write down the rule we are given, we are going to replace the variable with the number that we are given and then we are going to perform the calculation to get the answer. (Transcript of Lesson 3, October 30, 2012)*

Grace completed several examples on the whiteboard, changing the value of the variable each time. When Grace believed that the class knew and understood what to do, she assigned them some questions to attempt on their own. Towards the end of this lesson the class were working on translating worded problems into equations and using these to find unknown quantities.

*As we have just discussed  $35 \times 6$  because they want to hire the equipment for 6 hours.  $35 \times 6$  is 210 and then we are going to add 60, which means we get a total cost of \$270. This is a word question so you have to write your answer in context. (Transcript of Lesson 3, October 30, 2012)*

Finally, when Grace was satisfied that the students could complete similar questions, she broke the class into small groups to work on more questions, from the textbook. Grace asked students, in their groups, to come out to the front of the class to the whiteboard and explain how they had worked out the solutions to the set problems. This was a significant student contribution to the lesson as the students were not only working in their own groups, they were communicating their ideas to others and assessing others' work. In the task set, students had to write their own equations to solve a worded problem. There was the opportunity for students to make errors and to work with others.



*And you will work together and help each other explain it and then we will get you to come out to the board and show us how you worked these ones out.*  
*(Transcript of Lesson 4, October 31, 2012)*

In this final observed lesson, Grace's aim was to develop further the early algebra concepts she had been exploring with the class so as to continue to encourage students' thinking and their understanding of pattern and rules. The theme of practice used was Teaching/learning/student balance.

Table 34 *Teaching/learning/ student balance: Grace's theme of practice in guiding students' transition from number to algebra*

TEACHING/LEARNING/STUDENT BALANCE		
Code	Strategy	Orientation
10	Connection to prior learning	Connectionist
22	Models procedures and process	Transmission
14	Asks students for reasoning	Discovery
13	Engagement focus	Connectionist

From the Researcher's experience this is not a simple thing to do and would suggest that it is in the bank of strategies chosen and used by the teacher in this theme that satisfied well the aims of the lesson. The curriculum followed at Grace's school would have ensured that the students had had previous experience of continuing patterns, particularly with concrete materials. However, it was interesting to note that the teachers had not been able to access the rich bank of lesson plans available from the Maths 300 website, where many of the early algebra lessons make use of manipulatives. This will be discussed in more detail in the conclusion chapter of the thesis.

*I wanted to use Maths 300 with my students. Though we can get this software we've been battling to get the software loaded on to our laptops, and it is an IT department issue not mine so we can get it done in time. (Focus group Meeting, November 2012).*

### **6.3.6 Summary of Grace's knowledge of and beliefs about, the connection of established number (arithmetic) concepts with the effective teaching and learning of early algebra.**

- Grace is an experienced secondary teacher of mathematics.
- She has taught students from Year Seven to Year Twelve, the latter being at the specialist level.
- Grace enjoys teaching algebra to Year Seven and she believes that this can happen in an engaging and interesting way.
- Grace is confident in her ability to gauge students' readiness to learn algebraic language, conventions, rules and techniques.
- Her students' test results in the end of unit test support her belief that her students are learning and making good progress.
- Grace understands the conceptual shifts required for students to move in their understanding from concrete to abstract concepts in early algebra.

Grace employed strategies from the identified theme of practice of Teaching/learning/student balance in each of her four lessons.

In Lesson Two the Researcher observed Grace using this cluster of strategies to the maximum effect. It was evident that her use of questioning and her choice of tasks assisted students to make connections with prior learning. When new ideas or processes were introduced, she modelled procedures and practices. Reflection appeared to be used as an effective tool in her classroom and students were encouraged to think about what they were doing and to communicate their reasoning. Grace understands the need to engage students in what they are doing and her use of games and group work fostered participation by all students. As a teacher Grace believes that discovery is her preferred orientation to teaching and learning. She believes students should be encouraged to discover and investigate problem scenarios. However, the lessons observed by the Researcher did not concur with this professed belief.

Table 35 shows data gathered from the four lessons observed by the Researcher of the previously defined strategies. The percentage weighting of each in each lesson is shown in the table in the column called %W.

Table 35 Overview of Grace's observed practice

Lesson	Transmission	Connectionist	Discovery	%T	%C	%D
1	9	2	6	53	12	35
2	8	4	7	42	21	37
3	10	3	6	52	16	32
4	8	2	3	62	15	23

In three of the four lessons observed by the Researcher, more than half of the strategies used by Grace were those identified as being from a transmission orientation. Lessons One, Three and Four were weighted with transmission strategies, but in each of these lessons Grace also used strategies from a discovery orientation and those from a connectionist orientation. In Lesson Two Grace used a balance of transmission and discovery strategies facilitated by a number of connectionist strategies.

## 6.4 CASE 3: PENNY

Penny has been teaching for approximately eight years. Her teaching degree involved a science major with a mathematics minor. Penny works in the middle school of a large independent school in Western Australia. Students range from Kindergarten to Year 12. The school has a Year Seven transition program for entry into the secondary school. The school provides a wide range of courses. In 2012 the NAPLAN testing results in Numeracy showed that the number of students in Band 7 was statistically above the Australian average Penny has taught early algebraic concepts to Year Seven students for four years. Penny sees her class of Year Sevens four times a week; each lesson is of fifty minutes duration.

Table 36 which follows provides the reader with an overview of the content Penny covered in each of her lessons. The information is taken from notes the researcher compiled.

Table 36 Penny's lessons-Researchers notes

<p><b>Lesson 1</b></p> <p>This was the second lesson of an algebra unit. Recap on previous lesson. Introduction to terms, expressions, variables, pro-numerals and co-efficients. Focus of lesson writing algebraic expressions. Students work on questions from a worksheet begun in the previous lesson. Then practice questions from the e book. Mention made of equations and rule of order. Teacher works through the solutions to some of the questions the students have been doing. Students are given a page of notes to glue into their notebooks. Learning the language of algebra is the aim of the weeks lessons. Early next week they will start on equations. Mathematics tasks set up for homework. One more worksheet issued.</p>	<p><b>Lesson 2</b></p> <p>Information on structure given at the start of the lesson. Some bookwork, class discussion and a few class activities. Then students will practice what they have learned. Students take note on algebraic notation. Next exercise students copy table of terms on IWB. Students are asked to identify the pairs of algebraic expressions that are the same. This exercise brings out some misconceptions. This is a class task. Different ways to write expressions are modelled. Example, <math>n+n</math> is the same as <math>2n</math>. Students are asked to put their hands up if they agree or disagree with the solutions offered. Students given worksheet on algebraic notation. Random questions marked as a class. Meaning of indices revised.</p>
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<p><b>Lesson 3</b></p> <p>Focus of lesson to be writing algebraic expressions and how to simplify them. The work builds on the mathematics task the students have been working on. Introduction of the concept of Like terms. Review of terms, expressions, co-efficients, variables and pro-numerals. Worked examples modeled to students. Students are asked to write a short story which could represent the expression they are given. Students given notes. Students alerted to the fact they will need to remember the rules of integer operations. Students are given two worksheets. One on Like terms the other on simplifying. Students mark one then go onto the other.</p>	<p><b>Lesson 4</b></p> <p>Students need to download the application “Rover” onto their ipads. This allows them to use flash player features. Students work from website mymaths online. They are directed to work on a list of questions on algebraic manipulations and simplifying. When the students had completed the questions they moved directly to using their maths books. More notes are distributed and a worksheet. The questions deal with addition, subtraction and multiplication not division. Some questions are set for homework.</p>
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#### 6.4.1 Penny’s beliefs and practice for student development of number concepts

Penny made little mention of the laws of number in the lessons observed by the Researcher. However, she sees that a transition needs to be made from number to algebra in these early algebra lessons. The class had completed a unit of work on integers earlier in the year, prior to beginning the unit of work in algebra.

*I love it because for students it’s really the whole application of number. It seems that it is a point you are always working towards in maths, dealing with the unknown dealing with number and being able to use it effectively.  
(Interview 1, August, 2012)*

In the Researcher’s experience, assumptions can often be made about students’ understanding of the properties of number prior to beginning algebra. Penny believes that poor number skills should not hinder student learning in early algebra, and that it is permissible for students who struggle in this way to use a calculator.

*What sort of things do they really need to know? I am just going to cut my losses with this kid here. (Focus group meeting, November 2012)*

Penny recognised that students have ready access to calculators even on their phones so the question could be asked; Why bother learning to calculate without a calculator?

As the students worked on the practice questions on simplifying expressions, it became apparent to Penny that some students had forgotten the rules of number when dealing with negative integers. When dealing with multiplication and division in simplifying expressions Penny reminded students of their earlier work with integers and indices.

*People you are going to have to remember some integer rules. We tested you and we didn't say that you had to forget. (Lesson 4)*

Penny's view of the use of the calculator in an early algebra class is that it does not assist students. She suggested that having a calculator really would not make a difference if the student did not understand how to use algebra.

*In our particular program we actually had a no calculator test so when we did the algebra. We were more interested in them understanding the difference between like and unlike terms. You can have a calculator there. It's a tool there that you need to be comfortable to use. It's not really going to help you though. So we were not too interested in whether they can do two times three but more interested in the algebraic process and linking it to algebra. (Focus group meeting, November 2012)*

The Researcher would propose that indeed the calculator could be a very useful asset in the early algebra classroom, if not in a test situation. The calculator can be used to investigate how numbers behave under certain conditions. If a student does not have strong number skills it should not limit their exposure to these ideas. In the use of the calculator the student is still able to investigate number patterns and the laws of number. Incidentally the student may well improve their number skills. Further discussion on the relationship between number skills and success in early algebra is provided in Chapter Eight of this thesis, which

discuss limitations and conclusions. In analysis of Penny's practice in the four lessons observed by the Researcher there was insufficient data to identify a theme of practice in teaching number concepts.

#### **6.4.2 Summary of Penny's beliefs and practice for student development of number concepts**

- Penny sees that number has a role to play in students' learning of algebra.
- It is important for students to know the rules of integers.
- It is also useful for students to know what an index number represents.
- Penny believes that a calculator should be given to students who are struggling to understand early algebraic concepts, so that they are not precluded from learning because they do not have good number skills. However, she also believes that in working algebraically the calculator is of no assistance to students.

In the lessons observed by the Researcher there was no strategic attempt to develop students' concepts of number by Penny. Penny did remind students of the rule of order of operations and of the meaning of a number raised to a power, but these were small teaching moments within a lesson. Therefore, there was insufficient data to identify a theme of practice for Penny in the teaching of number concepts.

#### **6.4.3 Penny's beliefs and practice for students' development of early algebraic concepts**

Penny sees both the language of algebra and the rules and conventions of algebra as challenging for students to learn. These were two key objectives of her lessons. Penny also appreciates that the students in her class come with different levels of knowledge and experience of number and algebra.

*As it's their first exposure to it, it is trying to understand and trying to follow their work. Sometimes the students come in with a preconceived idea and you can build on it. Whereas for most students you are trying to teach it from*

*scratch and that ... in itself can be difficult. Just trying to develop the language, trying to develop the process. (Interview 1, August 2012)*

The three themes of practice Penny chose to assist students develop an understanding of algebraic concepts were of Teacher focus, Student focus and Teaching/learning balance.

TEACHER FOCUS		
Code	Strategy	Orientation
23	Use of practice in class	Transmission
36	Direct instructions	Transmission
17	Definitions presented and explained	Transmission
15	Whole class instruction	Transmission
STUDENT FOCUS		
Code	Strategy	Orientation
42	Allows student to investigate	Discovery
18	Students make their own notes	Discovery
35	Personalized Learning	Discovery
24	Follows textbook, worksheets	Discovery
25	Students work individually	Discovery
TEACHING/LEARNING BALANCE		
Code	Strategy	Orientation
34	Allows students to make errors	Discovery
47	Students work with others	Discovery
37	Closed questions	Transmission
40	Teacher assessment	Transmission
27	Connections made between ideas	Connectionist

Figure 26 *Teacher focus, Student focus and Teacher/learning balance: Penny's themes of practice in teaching algebra*

Penny's reflections on her practice and her participation in the focus group meeting signalled a change in the way that Penny viewed the teaching of early algebra and her understanding of how the students learn.

*Certainly a change I know I would like to make is to actually doing it a lot more*



*hands on instead of just being, here's a concept talk about it and get them to actually write it on the sheet. (Interview 2, November 2012)*

Penny expressed the intention of using more practical examples before moving to the abstract ideas. She still feels that the use of worksheets is important; however, it is providing students with tasks which engage their thinking that will guide her planning in the future. In Lesson One Penny asked questions of students to assess whether they had understood and remembered the previous work on the language of algebra.

*Can anyone tell me what the word left of the variable is called?*

Penny prompted the students' memories by talking about the work they had covered on using a formula to find the area of a circle. She pointed out the similarities between the work they did using a formula and their current task. In Lesson Three Penny advised the class on key ideas to note down, but the choice was still made by the students of the information they noted. The worksheet that students were issued with in this lesson advised them to look at the notes, which formed part of its content.

*My suggestion is read it first attack it with a highlighter. This is really your note page. You can add notes. (Transcript of Lesson 3, August 23, 2012)*

Penny then gave out two worksheets. One worksheet involved practice on grouping like terms and the second worksheet involved simplifying algebraic expressions. Students had to work through examples on the first sheet and mark their work before continuing on to do sheet two. Penny dealt with writing using algebraic notation primarily dealing with expressions in the second lesson observed by the Researcher. The students were directed as to which questions to attempt and how much time to spend on the task. Penny marked a random selection of questions with the class to assess their learning.

*I think it is time. I know a lot of you have had time to simplify with division and multiplication. We are not going to take time to mark all questions so instead we are just going to choose a few at random and go through the actual answers for them. So I would like you to trade your blue pen or pencil for a red*

*pen and we will mark only about 10 of the questions just to make sure you have been on the right track. (Transcript of Lesson 2, August 21, 2012)*

Also in this lesson Penny reminded students what it means to represent any number squared using algebra.

*a squared is the same as a times a. (Transcript of Lesson 2, August 21, 2012)*

Since being involved in this project, the Researcher would suggest that Penny is more aware of the limitations that a teacher encounters when the textbook becomes what they “teach to”.

*Putting a resource together is quite interesting because the textbook is the benchmark foundation on how you are going to deliver the content. I find it quite interesting. Different concepts don't actually suit all the students in the class, but one thing doesn't actually suit all the kids in your class. Using this worksheet will have half the kids are lost in the other half aren't quite there. (Focus group meeting, November 2012)*

In each of the lessons observed by the Researcher, Penny asked questions of students to assess whether they had understood and remembered the concepts they were learning. She also marked a selection of questions with the class as a whole to assess student learning. Penny set the students practice homework from an online mathematics website. In Lesson Four the class worked from another mathematics website where the students could test their knowledge and understanding of what they had been learning, and students were told that they could work with another person on these problems. However each lesson was taken as a class. The themes of practice Penny chose to assist students develop an understanding of algebraic concepts were Teacher focus, Student focus and Teaching/learning balance.

There were many opportunities in each lesson for students to practise their new knowledge.

*If I looked at my results, I guess my results were a bit lower than I expected*

*which was surprising. Students who were of lower ability in number all of a sudden did really well, were quite impressive. But I do find that students I thought had good problem solving skills didn't score as high as I thought. It definitely spread them out. From your well and truly below 50% to probably about your 70 to 80%. I didn't really get any to the top number I hoped to see I didn't get the full curve. I think it does come back to really what I was listening to in the discussion, interested in the style of teaching though, because I know a lot of it was sort of actually worksheet based and ok at the time like you said. They could actually recite all on the worksheet but then having that connection to it when it was time the test when things are phrased a little bit differently, that's when I think they struggled. (Interview 2, November 2012)*

When Penny was asked to write a reflection on the question, "Is there anything you would do differently? If you answer yes to this question can you elaborate by saying what you would change and why?"

*One of the things that I would like to do differently is to look at the way that the content was delivered. It was predominately taught using Powerpoint presentations, note taking, worksheets and online resources (Mathletics and mymathsonline). I would like to cover the same concepts but make it more student inquiry based and investigative. The reason for this is that I would like students to gain a better understanding of the applications for algebra and discover the rules of algebra through their own learning/experiences. (Interview 2, November 2012)*

#### **6.4.4 Summary of Penny's beliefs and practice for students' development of early algebraic concepts**

- Penny sees both the language of algebra and the rules and conventions of algebra as challenging for students to learn.
- Penny is aware of the differences in knowledge and understanding about number and algebra that students bring to the class.

- Penny did not use manipulatives in her teaching but by the end of the project she spoke of her intention to work with more concrete materials before moving to the abstract ideas.
- Penny spent a significant amount of her time, in the lessons observed by the Researcher introducing the language of early algebra and the conventions associated with calculations involving variables.
- Penny used worksheets for students to practise the language and rules they were learning in class and for homework.
- However, by the end of the project Penny changed her view of the use of worksheets. She acknowledged that worksheets have a role to play in helping students learn, but that their use should be balanced with other strategies.
- Penny encouraged students to make notes and to highlight important information on the notes she provided.
- The strategies used by Penny in assisting students to develop their understanding of algebraic concepts belonged primarily to the transmission orientation.

However, in each of the lessons observed in early algebra the themes of practice of Teacher focus, Student focus and Teaching/learning balance could be found.

#### **6.4.5 Penny's knowledge of and beliefs about, the connection of established number (arithmetic) concepts with the effective teaching and learning of early algebra**

Penny sees algebra as an important strand of mathematics and acknowledges the idea that it can be used to solve problems as its greatest strength.

*Absolutely, it is the problem solving element of it. It is about dealing with the unknown and actually looking at how to solve questions. (Interview 1, August 2012)*

In the first lesson observation, Penny encouraged students to view algebra in a positive way. She asked students to raise their hands to find out if their perception of algebra had improved by the end of the lesson.

*Hands up those who find algebra so far quite achievable, it's actually not that bad. (Transcript of Lesson 1, August 20, 2012)*

In the first interview Penny spoke of her experience in teaching algebra and her belief that by the time students reached Year Seven they are ready to learn about algebra.

*I find at that stage a lot of kids are wanting a challenge. I actually found a lot of kids are really quite receptive at this time. It definitely depends on the way you deliver it as I have learnt. (Interview 1, August 2012)*

Only in the first and fourth lessons observed by the Researcher did Penny use strategies identified as the theme of practice Teaching/learning/student balance. In each of these lessons Penny asked students to recall an idea they had previously been taught and from this thinking she framed her questions and modelled procedures to assist students to see the reasons for the new knowledge she was presenting them with. The students were engaged in the exercise and Penny gave students the opportunity to respond to her with their opinions, not just their answers to the questions she posed.

*Can anyone tell me where we have come across this idea when we looked at circles last term? Where have you already used algebra when we looked at circles last term? (Transcript of Lesson 1, August 20, 2012)*

*We are going to do a little online program today... Select rover on your ipad. I am taking you through the same process on the screen but I will be running it off my mac today. Some of things will be a little bit different so I will try and explain it as best as I can. Select your web browser. (Transcript of Lesson 4, August 27, 2012)*

The theme of practice in all of Penny's lessons was of Teaching/Learning balance.

Table 37 *Teaching/learning balance: Penny's theme of practice in guiding students' transition from number to algebra*

TEACHING/LEARNING BALANCE		
Code	Strategy	Orientation
34	Allows students to make errors	Discovery
47	Students work with others	Discovery
37	Closed questions	Transmission
40	Teacher assessment	Transmission
27	Connections made between ideas	Connectionist

In the final interview with the Researcher Penny revealed that her level of confidence in her teaching of early algebra had been reduced as a result of her reflections on her practice after the focus group meeting discussion. Penny saw that one positive outcome of her involvement in the project was the opportunity to meet the other teachers in the study. In the Researcher's first discussion with Penny about the aim of the study, she made clear her expectations. Penny hoped to learn from other teachers so as to continue to improve her knowledge and practice.

*So I might think I am not as good at teaching algebra as I thought, but hey I've got some good ideas and I want to investigate those. It's true, though that's one of the things about teaching. It is so fantastic to get other people's input because you want to challenge, you want to change to adapt. (Interview 2, November 2012)*

In the final interview Penny expressed her conviction that reflection was important to improve practice. She was aware that her observed lessons were heavily weighted with transmission strategies and worksheets.

*I think it does come back to really what I was listening to at the meeting. I know a lot of it was sort of actually worksheet based and ok at the time like you said they could actually recite all on the worksheet. But then having that connection to it when it was time for the test, when things are phrased a little bit differently, that's when I think they struggled. Certainly a change I know I*

*would like to make is to actually doing it a lot more hands on instead of just being here's a concept talk about it and to get them to actually write it on the sheet. I think the worksheets are probably one of the shortcomings of the ways I taught it. Rather than actually something that it benefitted. I would still want to use them though. It was too worksheet related though, so when it was time to be assessed if questions were a little bit outside what the worksheets had told them, they didn't have that true connection with that and I think that was probably one of the things I would like to change with the teaching. (Interview 2, November 2012)*

Penny also expressed concern about the issue of limited resources available in schools and that departments can become insular if they rely only on each other for ideas and strategies. As a consequence of her involvement in this study, Penny intends looking into more resources that will allow her students to investigate with more regularity in the course of their classwork. Penny sees that it would be helpful for teachers to have more support in terms of planning units of work in early algebra from more experienced teachers of mathematics.

*I think this distinction between what you're at and where you'd like to be. Allowing students to investigate that is something from me that I'd like to do more of. I'd probably like to investigate more resources. I could go on to the Internet and there are hundreds. It's almost nice to have someone who is a lot more skilled in the maths who can say these are the ones. (Interview 2, November 2012)*

When Penny had reflected on her practice she realised that this had actually not been true to her professed belief and she expressed her intention to make changes.

#### **6.4.6 Summary of Penny's knowledge of and beliefs about, the connection of established number (arithmetic) concepts with the effective teaching and learning of early algebra.**

- Penny sees algebra as important and its main purpose is to solve problems and to look for the unknown.
- Penny has taught algebra to Year Sevens for four years.

- The focus group meeting with the Researcher and the other teachers in the study and the Researcher proved for Penny to be a powerful learning episode for Penny.
- Penny expressed the intention to look for resources that would allow her students to make meaning for themselves, to investigate.
- When Penny reflected on her practice she believed that she had given students little opportunity to investigate the ideas that she was teaching, and that the worksheets she had used to assess student understanding had not proved to be a good source of information about the depth of student learning.
- Penny sees that more experienced teachers can be useful sources of ideas on the kinds of strategies that can be used to teach concepts in early algebra.

Table 38 shows data gathered from the four lessons observed by the Researcher of the previously defined strategies. The percentage weighting of each in each lesson is shown in the table in the column called T, C and D.

Table 38 Overview of Penny's practice

Lesson	Transmission	Connectionist	Discovery	% T	% C	% D
1	10	2	2	72	14	14
2	10	1	4	67	7	26
3	8	1	5	57	7	36
4	9	2	5	56	13	31

The majority of the strategies used by Penny in each of the observed lessons were identified as coming from a transmission orientation.



## 6.5 CASE 4: MARIE

Marie has been teaching for approximately seven years. She began her teaching career in England where, as a graduate, she completed a Post-Graduate Certificate of Education course with Cambridge University in teaching mathematics in secondary schools. This course has a large in school component.

*I was mentored in the school and I was pretty much teaching full time. Not a full time load but I was in the school full time. I received a lot of support from the school based prac teacher coordinator. I learnt a lot from on the job training. (Interview 1, June 2012).*

The school at which Marie taught in England was a mathematics and computing specialist college. After spending two years at the school Marie moved to Australia. Six months of relief work was followed by a full time position at her current school. During this study Marie has been promoted to Head of the Mathematics learning area at the school. The school is a large independent school in Western Australia. Students range from Kindergarten to Year 12. The school has a Year Seven Transition Program for entry into the secondary school. The school provides a wide range of courses. In 2012 the NAPLAN testing results in Numeracy showed that the number of students in Band 7 was statistically above the Australian average. When asked about her experience of teaching algebra she stated that she has been “teaching algebra at some level throughout her career”. (Interview 1, June 2012)

Table 39 which follows provides the reader with an overview of the content Marie covered in each of her lessons. The information is taken from notes the researcher compiled.

Table 39 Marie's Lessons-Researchers notes

<p><b>Lesson 1</b></p> <p>Question for the class on IWB. Spend some time exploring methods of long multiplication. Students start using ipads. Factor trees discussed. Lesson objectives written on the whiteboard. Writing expressions, expansion and factorizing. Arithmetic laws covered-Commutative, Associative and Distributive. Introduction to using symbols and numbers. Teacher uses manipulatives to explain expansion and factorizing. Students given papers with pairs of expressions some with or without brackets. Students then worked in groups to match expressions. The class do not complete the task and they will continue next lesson. Work completed is noted in journals. Homework set from text book. Students must do Q1. But can try more challenging questions which follow.</p>	<p><b>Lesson 2</b></p> <p>Students begin lesson by completing yesterday's task. Students record their completed work by taking a photo on the ipads. Then further work to consolidate the concepts of expansion and factorization. Students work with teacher on ipads. First with shapes and then with letters. Students could use a calculator. Students are required to explain expansion and factorization. Homework set. 20 minutes to be spent on questions from text book.</p>
<p><b>Lesson 3</b></p> <p>Students play Kim's game with numbers is on the IWB as a class group. Students had worked on mathletics in the previous lesson practicing expanding and factorizing expressions. Lesson focus on different ways of simplifying algebraic expressions. Students used "show me" on the ipad to answer questions. The teacher could see quickly which students were having difficulty. Some language of algebra was defined and noted by students. Co-efficients, Terms, algebraic expressions. Equations introduced. Students copy not from IWB on equations. Students task to sort equations and expressions into tables. Like terms explained before home work is set.</p>	<p><b>Lesson 4</b></p> <p>Students are in a different classroom. One with 4 whiteboards. When students enter the room they are given a slip of pink or yellow paper. Students work in groups to match the yellow and pink slips. One paper has the meaning in words the other the algebraic expression. Students prepare for writing equations by explaining the meaning of maths words like quotient. Students create expressions then equations from a phrase or a sentence. They then solve the equation. Student's work in small groups around the whiteboards in the room. This makes it easier for the teacher to find out who is struggling. Simple words problems are translated to equations. Students make their own notes in their journals. Students are advised of test date.</p>

### 6.5.1 Marie's beliefs and practice for students' development of number concepts

In the first of four lessons observed by the Researcher Marie began the lesson with a problem dealing with number operations. Marie asked students to investigate what happens when the same digits are used and reversed in a multiplication question. The order of the digits is changed. The problem projected on the board showed a cartoon character. Marie asked students where he would get the right answer even although he wrote the digits the wrong way round. She prompted students to think about the question: Would this work with any pair of two digit numbers?

*You can use calculators now for this. I actually want you to try and explore. Do you always get the same answer when you swap the digits or is it something special about those number? (Transcript of Lesson 1, September 6, 2012)*

Within this task students were also looking at different algorithms that they might have used in the past to complete two-digit multiplication. Marie posed questions as students tried various strategies to find a solution.

*What did he get? What should he have got and are there some numbers where that would actually work? Where he would get the right answer even although he wrote the digits the wrong way round. Is it ok to switch digits around? Try another couple of 2-digit numbers, your choice. (Transcript of Lesson 1, September 6, 2012)*

When students tried  $41 \times 28$  using a similar method to the one Marie used some students arrived at different answers.

*If you got different answers you had better figure out what has gone wrong. Should they be the same or different? So if you got different answers show the person next to you. See if you can figure out which step did you get wrong? (Transcript of Lesson 1, September 6, 2012)*

Marie asked students what they noticed about the numbers and they came to a conclusion about factors: numbers that can be broken down into factors and factors can be found using factor trees.

*Have any of you started on your investigation yet? So you have looked at factor trees. It is a bit like what we are doing now. We are looking at the factors of these numbers. Why is  $12 \times 7 \times 12$  the same as  $12 \times 4 \times 21$ ? Can I factor any of those other numbers, what do you see? (Transcript of Lesson 1, September 6, 2012)*

Marie made explicit the associative thinking for students between the operations  $14 \times 2$  and  $14 \times 80$  and the calculation required for  $14(80+2)$  (using the distributive law). This teaching point was a major objective of the lesson. The students themselves made the connection between the number task they had started with and the algebra they finished the lesson using. Marie was aware that the Year Eight class of students she would be working with had struggled with mathematics in the previous year. She therefore treated the class as a Year Seven class and planned her lessons accordingly.

Marie was concerned when examining Lesson Six (Appendix 8) at the focus group meeting that the proposed lesson starter of a quick quiz could be a problem for some students. The quick quiz comprised 10 mental arithmetic calculations. Her concern arose because she believed that some students would be unable to attempt the questions because of poor number skills.

*I take the point to looking at that lesson three you've got your quick quiz questions there. You would have students in Year Seven who would not be able to do that and you as a teacher have to decide well how much time do we spend on this before progressing on to the lesson. To have a task that you can then differentiate. (Focus group meeting, November 2012)*

In teaching for students' development of number concepts Marie used strategies from the three themes of practice of Student focus, Teacher focus and Teaching/learning balance.

STUDENT FOCUS		
Code	Strategy	Orientation
42	Allows student to investigate	Discovery
18	Students make their own notes	Discovery
35	Personalized Learning	Discovery
24	Follows textbook, worksheets	Discovery
25	Students work individually	Discovery
TEACHER FOCUS		
Code	Strategy	Orientation
23	Use of practice in class	Transmission
36	Direct instructions	Transmission
17	Definitions presented and explained	Transmission
15	Whole class instruction	Transmission
TEACHING/LEARNING BALANCE		
Code	Strategy	Orientation
34	Allows students to make errors	Discovery
47	Students work with others	Discovery
37	Closed questions	Transmission
40	Teacher assessment	Transmission
27	Connections made between ideas	Connectionist

Figure 27 *Student focus, Teacher focus and Teaching/learning balance: Marie's theme of practice in teaching number*

Marie sees engaging all students in learning in a lesson as important. To do this the task must be something that all students can attempt at some level. Ensuring that the students have the pre-requisite skills is therefore essential, and evidence of this is something the Researcher observed in Lesson Four. The main aim of the lesson was to enable students to translate a worded phrase to an expression and a worded statement to an equation. To do this the students needed to understand the meaning of some words commonly used to describe number operations. In the Researcher's experience the incorrect assumption can be made that students know and are familiar with this language. Just like the language of early algebra the language of number operations can be misunderstood or forgotten.

*When we had the lesson on difference quotient I could see straight away all*

*around the room who had got it. (Interview 2, November, 2012)*

When asked by the Researcher when the students would have learned about integers Marie response was:

*They would have done that this year and last year and all the way through primary school but they were of quite a low ability group and when they were in Year Seven they didn't have a specialist maths teacher so I haven't made any assumptions of them being able to do any of that work. I treated them as if they were Year Sevens. (Interview 2, November 2012)*

In her reply Marie raised two important issues. Why do some students fail to learn when they cover the same ideas year after year? And how important is it to have a specialist mathematics teacher in Year Seven? It is not the purpose of this research project to explore these issues but it is worth noting them at this point.

In the first lesson observed Marie also introduced the students to the definitions of the properties of real numbers. She explained the meaning of commutativity, associativity and the distributive property, giving students examples of each.

*So this first property is called commutativity. Have you heard of that word before? It is a big long word, which means that with certain operations you can swap the numbers round.  $11 + 4$  is  $4 + 11$ . So what else is commutative? Addition is commutative. What else have we shown today? Multiplication is also commutative*

*Another property is called Associativity So these things are associative which means we can add the 4 and the 3 which gives 7 and then add to 9 makes 16 or we could add the three and the nine, then add 4. Means you can group in different ways and still get the same answer.*

*This last property is really what we have been doing with expanding  $6 + 2$ , everybody 8. Five lots of ... 40. I can separate the 6 and the 2 or expand the 6 and the 2. Five lots of 6 and 5 lots of 2 is also 40. Let me write this over here. Five lots of 6 plus 2 means 5 lots of 8, which is 40 forty but I can do that*

*separately. Five lots of 6 plus 5 lots of 2 is 30 plus 10 also 40. That property is called the distributive property and it is the same thing really as what I was talking about earlier with expanding. (Transcript of Lesson 1, September 6, 2012)*

### **6.5.2 Summary of Marie's beliefs and practice for students' development of number concepts**

- Marie prompted the students to think about the way that they might carry out a two digit multiplication as a starter problem.
- Marie clearly defined the laws of number: associative, commutative and distributive. She then modelled their use in worked examples with the students.
- Prior to the students completing work on creating expressions and equations, Marie defined for the students the meanings of common words used to describe calculations in the mathematics classroom.
- Marie did not assume that students in her class would know the meanings of the words they were about to be asked to use in the task of translating worded phrases and statements into algebraic expressions and equations.
- Marie wanted students to make the connection for themselves between the ideas they had been considering in number with what they were being asked to do in algebra.
- In naming and explaining the laws of number Marie hoped that students would see these ideas transfer to how they think about variables and operations, which included variables.
- Using a balance of strategies from a transmission (Teacher focus) and discovery orientation (Student focus), Marie's use of the cluster of strategies from the Teaching /learning focus provided students with the structure in which they could make sense for themselves.

In teaching for students' development of number concepts Marie used clusters of strategies from three themes of practice, namely Student focus, Teacher focus and Teaching/learning balance.

### 6.5.3 Marie's beliefs and practice for students' development of early algebraic concepts

*I love teaching algebra. It is very enjoyable. It is challenging to be able to pitch it at the right level. I think that is one of the reasons teachers and students get turned off algebra because too much is taught too soon. I think that sort of panic set in a bit here when we realized they would need to teach algebra to Year Seven. I have observed colleagues doing quite complex simplifications of expressions and I have sort of observed that and thought no that is too much. You've got to really start at a basic level. But for me that is part of the enjoyment challenging to do that. When they really click with it and it suddenly makes sense there is a satisfaction with that. (Interview 1, June, 2012)*

Marie enjoys teaching algebra. Her positive approach set the tone for the lessons and students appeared to engage fully with the work, undeterred by the presupposed ideas they may have had about the difficulty of algebra. One of the significant challenges Marie sees that teachers face is that of presenting the early algebra concepts to students at the appropriate level. This notion of pace is important to Marie and she restated her concern in the interview with the Researcher.

*That kind of knowledge of what students are able to do and to be able to recognise a student is ready to progress and it doesn't necessarily happen at the end of Year Seven going into Year Eight. It has got to be tailored to those students. I suppose another hurdle would be at the other end of the teaching spectrum where you have got teachers who are not qualified mathematicians but because of the shortage of expertise that they end up maybe they have got a maths minor or something like that and so they are not necessarily comfortable with the subject matter. They learn a procedure but don't really understand or have a sense of the progression. (Interview 1, June 2012).*

In teaching for students' development of algebraic concepts Marie used clusters of strategies from the three themes of practice of Student focus, Teacher focus and Teaching/learning balance.



STUDENT FOCUS		
Code	Strategy	Orientation
42	Allows student to investigate	Discovery
18	Students make their own notes	Discovery
35	Personalized Learning	Discovery
24	Follows textbook, worksheets	Discovery
25	Students work individually	Discovery
TEACHER FOCUS		
Code	Strategy	Orientation
23	Use of practice in class	Transmission
36	Direct instructions	Transmission
17	Definitions presented and explained	Transmission
15	Whole class instruction	Transmission
TEACHING/LEARNING BALANCE		
Code	Strategy	Orientation
34	Allows students to make errors	Discovery
47	Students work with others	Discovery
37	Closed questions	Transmission
40	Teacher assessment	Transmission
27	Connections made between ideas	Connectionist

Figure 28 *Student focus, Teacher focus and Teaching/learning balance: Marie's themes of practice in teaching algebra*

Marie sees there is a real need for teachers to be aware of the mathematics learning continuum and the mathematics history or experience of the students they are teaching. The curriculum can then be tailored to meet students' learning needs. It should not be the case that because a student is in Year Eight that they should necessarily be working on algebraic concepts specifically designated as Year Eight. These students may well need to revisit earlier concepts if they are to make sense of these and what follows next. This was the situation with the class Marie taught during this study. In her Year Eight class she was working on the same ideas as the three Year Seven classes in the study. Marie sees the transition from letters to numbers as a large conceptual leap for students to make.

*This whole idea of, why are we working with letters instead of numbers? that conceptual grasp of letters as symbols, I think that takes a lot.  $3a$  when  $a$  is 7 a student can read as 37 because they don't grasp the concept. It is just not writing the number down. That is a difficult concept. (Interview 1, June 2012).*

Marie sees that negativity that parents and teachers may feel about their own experience of algebra can impact on students' learning.

*I think also a lot of students that their parents will say I didn't like algebra; I was no good at algebra. So there is a culture they pick up even from other teachers that algebra is hard, it's confusing and irrelevant and those ideas get thrown around. (Interview 1, June 2012).*

In scaffolding students' learning Marie set out to ensure that her students understood the algebraic concepts of variable, term, expansion, factorization, expression, and equation. Marie presented these ideas several times in different contexts throughout her lessons. In the first lesson observed by the Researcher, Marie distributed some strips of coloured card to groups of students. On these domino strips were written algebraic expressions some of which had been factorized and some of which had not. The students were asked to match the pairs of expressions.

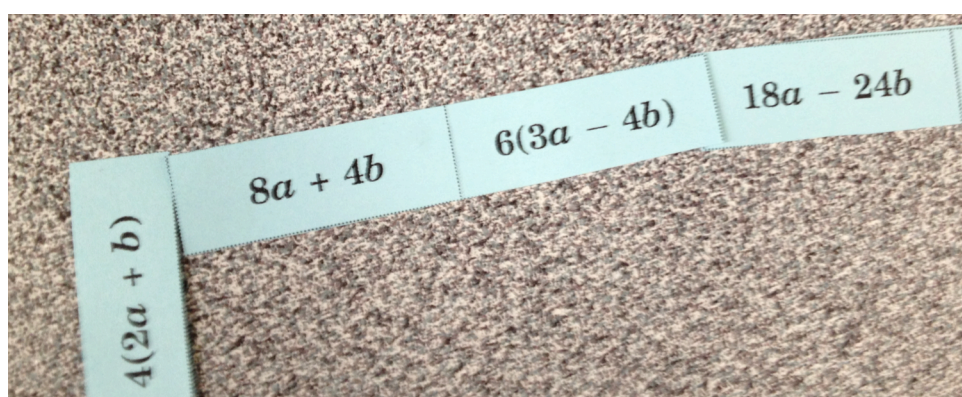


Figure 29 Student work

Students did not complete this task in Lesson One, and so resumed their work on this in Lesson Two. They recorded what they had found to date before the end of the first lesson.

*I want you to write any pairs that you found down so that we can continue with this tomorrow. Everyone writes it down not just one person. (Transcript of Lesson 1, September 6, 2012)*

This task appeared to engage all of the students. Samples of the notes that the students made are provided in Appendix 11. Marie ended the lesson by setting the class some practice questions for homework from their ebook text. Students were advised that they could do more if they wished, but that there was a minimum requirement. In this way she differentiated the homework activity so as to personalize their learning.

In the second lesson observed by the Researcher, the students worked on a further investigation of the concepts of expanding and factorising expressions. Marie asked the students to sketch shapes on their ipads, in the manner that they had learned in Term One. In this lesson the emphasis was helping students transition from what they had been doing with number and shape to using numbers and letters.

*On your "show me's" can you please show me how to simplify or write that in a different way? Preferably a simpler way.*

*Some interesting responses. Quite a few people wrote 3 box and some people wrote box with a little three like that so what do you think are they both correct? Yes, no what do you think?*

*Talk to the person next to you. I want you to have a bit of a discussion about what each of those things mean. What does this mean? What does that mean?*

*Talk to the person next to you. What does 3 box mean and box with the little three mean? (Transcript of Lesson 3, September 11, 2012)*

In this questioning Marie was asking the students if they understood the difference between multiplying an unknown number by three or raising the number to the power of three. She posed a question to the class and then asked them to vote on it by responding with an identified gesture. This allowed Marie quickly to see those students who did not understand the concept of an index number.

*I have moved into the domain of  $x$ 's and  $h$ 's, lets see what you think.*

*When we put letters and numbers together in algebra what does that mean?*

*It is confusing to people who are new to algebra because all the way through primary school you see a symbol like add and you think you have got to do something else. It doesn't look finished, does it? In algebra it is ok to have an addition sign in your answer. (Transcript of Lesson 3, September 11, 2012)*

Marie also did some further work with the students on algebraic expressions. She projected some examples on the whiteboard and asked students to think about how many terms were in each. Marie reminded students of the difference between expressions and equations by giving examples that she then discussed with the class.

*We have looked at terms, we have looked at expressions which are groups of terms. The last bit of notation for today is equations. What is it that makes an equation? (Transcript of Lesson 3, September 11, 2012)*

Writing in their journals Marie directed the students to draw up a table with the headings terms, expressions and equations. Their task was to complete the table with the correct item from the list on the whiteboard. Students were advised that they could make up some of their own. In observing the students completing this task, Marie could assess whether the students could identify the objects correctly.

In the third lesson observation the Researcher found that Marie had spent some time in with the class practising what they had learned about algebraic simplification, using the computer website Mathletics. Marie reminded students of the language of algebra and defined the words coefficient, expression, variable, terms and like terms giving examples of each in turn.

Marie believes that teachers should be reassured by the advent of the Australian Curriculum and by what she sees as the freedom it affords teachers as to the degree of difficulty they choose in regard to the concepts they teach

*I think there's a perception that there is a lot more in it and it has panicked some teachers. And certainly the introduction of certain particular things in*

*algebra when you look at their national curriculum itself it's quite sparse and there's a lot of freedom as to the degree of difficulty you then take these ideas which obviously depends on the students you are working with. (Focus group meeting, November 2012)*

For the final lesson observed by the Researcher the class worked in a different classroom. The classroom Marie had chosen made it easier for students to move around. The classroom also had large whiteboards, which made it possible for the students to work together in a visual way when required. In this lesson she provided students with examples from which they made their own notes. When students entered the classroom they were directed to choose a strip of coloured paper the teacher held out to them at the front of the classroom and to find a seat. Marie told students to seek others who had equivalent expressions and to sit with them, and she used this activity to remind students of the work covered in previous lessons. Marie informed students that the purpose of today's lesson was to look at building equations and solving them.

In this lesson Marie also wanted the students to understand the language they would see when constructing an expression from a phrase and an equation from a worded statement. She projected the list of words onto the board. Marie explained the meaning of each in turn as the objective of the lesson was for students to be able to write both expressions and equations. Throughout the lesson Marie was clear about defining the meanings of the words and that they would see be using in writing phrases and sentences.

*Before we do this activity just quickly grab your journals, let's make some notes on this and then I can erase that from the board. You need to make notes on those things and what they mean. (Transcript of Lesson 4, September 21, 2012)*

Many teachers, Marie suggests, are guided too much by the chosen textbook and not the curriculum documents.

*I think that's one of the issues with a textbook that the minute you put a textbook in front of a teacher many feel that it's the aim of the game is to get through the textbook. The curriculum is not the book. The book is one of the many tools. (Focus group meeting, November 2012)*

As Marie reflected on her own use of the class text throughout the unit of work she indicated that change had occurred and would continue to occur.

*I have said in the questionnaire as well, but because we've asked the students to purchase a textbook and it's an e-book there is a sense that we feel sort of feel obliged to use that book. In actual fact the progression in this topic was not really suitable for my students particularly things like negatives coming in earlier when they were really just getting their head around simplifying and factorizing, expanding that sort of thing. So I think I would be a lot more selective about when it was time to do practice questions. In actually selecting appropriate questions for the level and keeping the harder ones in reserve for students who are ready for the challenge. (Interview 2, November 2012)*

Marie attributed student success in the end of unit test to their engagement in classroom activities and to the belief that she had encouraged, that they could learn algebra. She also suggested that in ensuring that the test assessed what the students could do, all were able to gain some sense of achievement from the experience.

*I also think we were a bit more careful when we put the test together. As I said at the beginning of the research you know there's a temptation to want to expose them to every possible, you know negatives and complicated simplification and whatever. (Interview 2, November 2012)*

#### **6.5.4 Summary of Marie's beliefs and practice for students' development of early algebraic concepts**

- Marie particularly enjoys teaching algebra.
- She sees that it is a challenge for some teachers because they try to teach too many concepts together, and at a time when students are not ready to learn them.
- Marie believes that if teachers of early algebra are not qualified as teachers of mathematics or have no real understanding of the progression of thinking required themselves then there is the danger that they will try

to teach too much too soon.

- Marie's class was made up of Year Eight students who had struggled with number and algebra in Year Seven. Marie worked with the class as a Year Seven class and she tried to make no assumptions about the students' knowledge, understanding and skills in number and algebra.
- Marie wanted students to make the connection for themselves between the ideas they had been considering in number with what they were being asked to do in algebra.

Using a balance of strategies from a transmission (Teacher focus) and discovery orientation (Student focus) Marie's use of the cluster of strategies from the Teaching /learning balance provided students with the structure to have this new knowledge make sense for them. In teaching for students' development of algebraic concepts Marie used clusters of strategies from the themes of practice of Student Focus, Teacher Focus and Teaching/learning balance.

#### **6.5.5 Marie's knowledge of and beliefs about, the connection of established number (arithmetic) concepts with the effective teaching and learning of early algebra**

Marie is confident in her ability to teach students for the transition from number to algebra. This may well be due to the intensive introduction to teaching practice she received in her Cambridge course, supported by her experience as a mathematics graduate.

*We don't need to be scared about the national curriculum which says we have to do algebra with Year Seven because actually what the national curriculum says is quite sparse and the degree of difficulty that we go to is dependent on our students and our professional knowledge. (Interview 2, November 2012)*

Although Marie has yet to teach a Year Seven class in Australia, she has worked with a group of talented Year Sevens on problem solving activities. Marie experienced teaching Year Sevens in England. Her major recollection of the work done with Year Seven at that time was that the teaching program had much less emphasis on assessment than she has seen in Western Australia.

*There was much less emphasis on testing. Our emphasis was about making Year Seven mathematics enjoyable. Less on assessment very creative, very collaborative, using ICT I don't remember doing any assessments with this group. Other than optional standardised tests. Very, very different from now. (First interview, June 2012)*

Marie sees algebra as an important strand of mathematics.

*Yes. Algebra is an important strand of mathematics. The thing with algebra it is where students are exposed to generalising, where they are able to summarise. (Interview 1, June 2012).*

She sees that algebra is much more than a series of rules, techniques and manipulations.

*Algebra is another language. You see the world through that you can describe patterns. You can describe relationships using the language of algebra.*

*To me it is like looking at Art. (Interview 1, June 2012).*

When given a copy of the planned unit of work on early algebra (Appendix 9) Marie observed that the resources listed came from a range of sources. This had indeed been an important consideration in its compilation by the Researcher.

*I really like this idea in principle, you know the fact that you're drawing on lots of different resources and looking at the national curriculum. So many units of work are just, here's a textbook exercise 1.1 followed by exercise 1.2. This, sort of looks like all of the building blocks. (Focus group meeting, November 2012)*

Marie is familiar with the Maths 300 (Education Services Australia, 2010) teaching resources that she has access to in her school. She also has access to the mathematics task centre resources. Marie suggests that the challenge for teachers can be in finding the time to sort through all the material in their busy schedules.



*I think I was already fairly confident but I think I've now got a few more new ideas and resources I can bring out particularly with weaker students and also just conversations with colleagues both within and outside of the school.*

(Interview 2, November 2012)

Marie sees herself as a teacher who believes that students learn best when they discover ideas for themselves.

*The best moments in my teaching has been moments when students have discovered things for themselves not when I have told them how to do something. (Interview 2, November 2012)*

In her responses to Questionnaires I and II, and also in her practice, Marie appears to give almost equal weight to transmission strategies. This may be attributed perhaps more to her beliefs about the nature of mathematics itself, rather than the teaching and learning of mathematics.

Marie's use of transmission strategies in these early algebra lessons, the Researcher would suggest, is directly linked to the nature of the work being covered. In these lessons students are being initiated into the language and conventions of algebra.

*My heart's definitely leading to the discovery and there are certainly elements of that in a lot of my lessons but I wouldn't like to put myself out there as a discovery teacher because I know a lot of the time it's transmission. (Focus group meeting, November 2012)*

In the final interview with the Researcher Marie spoke of her awareness of the different strategies she used and her beliefs about learning. Marie was concerned that in her teaching the balance of strategies used did not reflect her beliefs, and the Researcher would concur with her assessment. However, the Researcher would suggest that the data gathered indicates that Marie is a teacher who uses a *balance of strategies*: that is, she used strategies from each of the three orientations in her lessons in early algebra.

*I get it that both have value and I'll still be including both. But I would say again talking to colleagues in and out of the school. We as practitioners are not in danger of using not enough transmission I think it is usually more the other way. We need to work hard at bringing in discovery activities. We always revert to transmission when we are stressed when we are pushed for time and we can't think of another way to do things and we think there is a test coming up they have to know this stuff therefore I have to convey this knowledge to them. So that concept of less is more just goes out the window. We just end up of trying to get them to do these questions the way I have told them to do. (Interview 2, November 2012)*

The theme of practice in all of Marie's lessons was Teaching/learning/student balance in guiding students' transition from number to algebra.

Table 40 *Teaching/learning/student balance: Marie's theme of practice in guiding students' transition from number to algebra*

TEACHING/LEARNING/STUDENT/BALANCE		
Code	Strategy	Orientation
10	Connection to prior learning	Connectionist
22	Models procedures and process	Transmission
14	Asks students for reasoning	Discovery
13	Engagement focus	Connectionist

*Teaching/learning/student balance: Marie's theme of practice in guiding students' transition from number to algebra*

In the first lesson observed by the Researcher the puzzle at the start of the lesson was such that  $14 \times 82$  should have been the problem, but the cartoon character made a mistake and wrote down  $41 \times 28$ . In this lesson Marie asked students to think about what happens when digits are reversed. The students were encouraged to explore different possibilities, making connections to prior learning. The task was open to all and achievable by the students in the class. The students could work on paper or on their ipads. One student worked on the whiteboard at the front of the room. Students were asked for the method they used and the thinking behind their actions. Marie modelled the process of the operation of long

multiplication, seeking students' input starting with the familiar algorithm  $14 \times 2$  and  $14 \times 80$ . Marie asked the class to think about how this method works, and also to think in the same way about the method they may have used. In Lesson One, Marie made the transition from using the distributive law with number and moving to algebra both interesting and memorable for her Year Eight class by using manipulatives. These manipulatives were bags of tiny teddies biscuits and bags of milky bar chocolate buttons.

*Now it is going to get interesting because we are going to use some symbols as well as numbers. You've done a little bit of algebra before this so this is  $x$  here is a symbol and it represents a number. I have brought some props today to help me with my symbols. So you are going to associate for evermore algebra with tiny teddies and milky bar buttons, positive associations with algebra. (Transcript of Lesson 1, September 6, 2012)*

As there was no way of knowing how many tiny teddies there were in a bag and assuming that there was the same number in each bag Marie assigned the letter  $x$  to represent the number of tiny teddies in a bag. Marie emphasised that  $x$  represented an unknown number. The unknown number was the number of tiny teddy biscuits in the bag. MacGregor & Stacey, (1997) found along with many other Researchers that there is a real danger of students having the misconception of a letter representing an object. Throughout this lesson Marie reiterated that a letter represented an unknown number. Marie directed the students to look at the expression  $3(2x + 4)$  on the board.

*So we have got two lots of tiny teddies in bags plus an extra four tiny teddies, so that is symbolised by  $2x + 4$ . Now the 3 means I am going to do that three times. (Transcript of Lesson 1, September 6, 2012)*

Marie guided class discussion to assist students come to the conclusion that this expression could also be written as  $6x + 12$ . She demonstrated this using the bags of tiny teddies and some loose biscuits and wrote the expansion in full on the whiteboard. Marie gave students more examples of writing algebraic expressions in different ways, in this case expanded and factored, using the variable  $y$  and the number of bags of milky bar buttons.

*Can you see now that 4 lots of  $3y + 2$  is the same as 12 lots of  $y$  plus 8? Is this making sense to you? You don't need to draw a diagram just write down that equation for me so we can refer back to it. (Transcript of Lesson 1, September 6, 2012)*

Marie began the third lesson observed by the Researcher with a version of Kim's game (a memory game). A complete table of number facts are was displayed on the board and then the number facts disappeared one at a time. This activity proved to engage students and encouraged them to focus. Marie then moved to remind students of their previous work on expansion and factorising algebraic expressions. Using their ipads and the application named "Show Me", Marie asked the students to show her how they would simplify or write the expression she had on the whiteboard in a different way.

She received some interesting responses from students. More than one response showed that there was some confusion between the  $3x$  and  $x^3$ . Marie asked the students for their reasoning.

*Talk to the person next to you I want you to have a bit of discussion about what each of these things mean. What does this mean? What does that mean?*  
(Transcript of Lesson 3, September 11, 2012)

Marie gave students the opportunity to change their answers when the meaning of the index became clear.

Marie was pleased with the level of engagement by students in the fourth and final lesson observed by the Researcher. In the final interview, when she reflected on her practice, she spoke about the positive learning outcomes for students and her intention to try more activities of a similar mode in the future.

*Also when we had the use of the room with the whiteboards all around and students were working collaboratively, working out their answers together, discussing it even though it was really you know, the questions were no different from if they were sitting down at a desk working individually. The fact that they were up at the board increased the engagement and also because*

*they were having to verbalize and collaborate on the tasks. I felt that their understanding was increased by that, so I feel freer to try different things rather than just sitting at the desk doing practice questions. (Transcript of Lesson 4, September 21, 2012)*

#### **6.5.6 Summary of Marie's knowledge of and beliefs about, the connection of established number (arithmetic) concepts with the effective teaching and learning of early algebra.**

- Marie is confident in her knowledge of content, curriculum and pedagogy.
- Her own knowledge of mathematics and the training she received has supported her as a teacher.
- Marie sees algebra as an important strand of the mathematics that has its own inherent beauty.
- It is a language with which to communicate, and to describe relationships and pattern.
- Marie is familiar with a variety of resources that can be used to teach algebra.
- She is eager to provide students with opportunities to discover ideas for themselves, which she believes is the best way for students to learn.
- In practice this was not the case. Marie became aware of the difference and her goal is to work to be truer to what she believes in her practice.
- Marie believes that students learn best when they discover for themselves, and in her lessons she used strategies defined as originating from a discovery orientation. However, those strategies were often balanced with others defined as originating from a transmission orientation. Between the two orientations she used strategies defined as originating from a connectionist orientation.

The theme of practice in all of Marie's lessons was of Teaching/learning/student balance.

Table 41 shows data gathered from the four lessons observed by the Researcher of the previously defined strategies.

Table 41 Overview of Marie's Practice

Lesson	Transmission	Connectionist	Discovery	% T	% C	% D
1	10	5	9	42	21	37
2	5	4	10	26	21	53
3	8	4	8	40	20	40
4	4	5	7	25	31	44

Marie believes that the use of a variety of resources encourages student engagement. Marie's class of Year Eight students had had little success in the previous year when they had first been introduced to algebra. Marie encouraged students to engage with the ideas in her lessons.

## 6.6 Chapter Summary

In this chapter the analysis of qualitative data gathered from interviews, lesson observations, the focus group meeting and personal reflections are presented in narrative form to construct each individual case within the larger case.

Initial analysis of early quantitative and qualitative data gathered provided the foundation on which aspects of the teachers' PCK was then explored. Observed practice was then analysed.

In the next chapter, the cross case analysis is presented. A discussion of the findings in relation to the literature, the answer to both research questions, including also the limitations of the study and recommendations for future research, are to be found in Chapter Eight of the thesis.

# Cross Case Analysis and Discussion

## 7.1 Introduction

In this chapter the professed beliefs and the actual practices of the four teachers in the study are compared and contrasted. Findings discovered through analysis of aspects of the teachers' PCK (pedagogical content knowledge) are supported by the literature. The analysis of the teachers' practice was founded on the three underlying teaching orientations of transmission, discovery, and connectionist (Askew et al., 1997; Swan, 2005), discussed earlier in the thesis. This work contributes to the body of knowledge concerning teaching for the transition from number to algebra by providing a rich exploration of the teachers' practices and in particular the strategies used by the teachers in the Year Seven and Year Eight classrooms in which they worked. The teachers' practices were examined in the light of their professed beliefs about the nature of mathematics learning and teaching identified by the quantitative data analysed and presented in Chapter Four of this thesis.

## 7.2 The Participants

The four teachers in the study were willing to share their time, knowledge and personal experience teaching early algebra in order to further the collective understanding of teaching in early algebraic concepts and the beliefs that underscore teachers' actions. All four teachers taught at low fee paying independent schools in Western Australia. There were many similarities in the school cultures, and the classrooms visited were places conducive to learning. Facilities in all four schools were well established and teachers and students had access to computers and interactive whiteboards and or data projectors. In two of the four schools students used personal ipads in classes. In one school students had their own personal laptops and in another school students had access to computers from a bank of laptops or by visiting a computer lab. There were differences in the classes. Two classes of Year Seven students were un-streamed and one class of Year Sevens was a streamed class of top students, this was Grace's class. The fourth class was Marie's class of Year Eight students who had struggled with algebra in Year Seven. At the beginning of the study all four teachers were confident in their level of knowledge and their ability to teach early algebra concepts. The teachers' understanding of the development of number, the nature of algebra and their own

content knowledge of teaching early algebra were identified by the Researcher as forming part of their pedagogical content knowledge as teachers of mathematics. The teachers in the study had varying degrees of experience in teaching algebra to Year Seven or Eight students. The teachers' knowledge of how to teach algebra was predicated by their own personal study and experiences as teachers of mathematics and in some instances influenced by the textbook used for the course. Concern about the latter was raised by Marie at the focus group meeting and in an interview.

### **7.3 Comparisons of the Knowledge and Dispositions concerning the Learning and Teaching of Algebra**

At the beginning of the study all four teachers were confident in their own pedagogical content knowledge for teaching for the transition from number to algebra. As the study progressed this changed. Marie, Grace and John spoke of increased confidence. After reflection they believed that their work had been validated by their students' engagement and success in end of topic tests. In contrast, Penny spoke of a decrease in her level of confidence. Each of the four teachers worked to engage students and encourage interest in early algebra by explicitly defining what they saw algebra to be and the importance of the ideas in algebra. The teachers also devised activities for the students which were inclusive and demanded action by the students.

Each of the teachers in the study saw algebra as important, although their reasons for doing so were different. Marie saw not just the working usefulness of algebra to solve problems but the beauty that can be found in algebra for its own sake. She emphasized the importance of appreciating the patterns and the creativity. John saw pattern as important and he encouraged his students to look for mathematical pattern not just in a lesson on algebra. Grace mentioned the practical uses of algebra and where it could be used in other subjects. She also highlighted for students when a foundational idea was introduced and how it would be built on in future years in the mathematics classroom. Penny saw algebra as a means to solve a problem, to make hard things simpler, thus her emphasis in the lessons was to have students practise simplification of terms.

The Researcher examined the content each teacher was teaching in this transitional period. The question of the order in which different mathematical competencies should be



developed has always drawn the attention of psychologists and educators. Sousa (2008) has suggested that moving students from the use of concrete materials to abstract thinking is in essence a movement from number to algebra.

Table 42 Overview of concepts in beginning algebra covered by each teacher in the four lessons

List of Items taken from the statement of Year 7 achievement standard ( <i>The Australian Curriculum pp. 49,50</i> )	John	Grace	Penny	Marie
Addition and subtraction of integers	<i>Briefly reviewed</i>	Prior to study	Prior to study	Prior to study
Index notation	○	○	✓	○
Represent numbers using variables	✓	✓	✓	✓
Connect laws and properties for numbers to algebra	<i>Not explicitly</i>	<i>Not explicitly</i>	<i>Not explicitly</i>	✓
Interpret simple linear representations	✓	✓	✓	✓
Solve simple linear equations	✓	✓	○	✓
Evaluate algebraic expressions after numerical substitution	✓	✓	✓	✓

Only one teacher taught the laws of number as a precursor to the introduction of variables. This is discussed in the next section of this chapter.

#### **7.4 Comparison of the Teachers' Knowledge of, Beliefs and Practice concerning the Teaching of Number Concepts**

Marie was the only teacher who explicitly taught the laws of number. She also made a clear link between the properties in number and how these transfer to algebra. The other three teachers in the study made some mention of the rules of number in calculations, but these were incidental and not a lesson objective. This matched some perceptions of Researchers as to the lack of preparation in number (Warren, 2003). At the focus group meeting and in final interviews it became apparent to the Researcher that John, Penny and Grace intended in the future to lay greater emphasis upon explicit teaching for the transition from number to algebra, by exploring the laws of number as early algebra instruments. Grace had not seen the need with the selective class she had taught but with a different class this would change. The four teachers in the study included the use of integers in their lessons and the Researcher would suggest that it would be difficult to find a Year Seven or Eight text which did not use negative integers in beginning algebra exercises. It was not surprising to find that students in all four classes had to be reminded about how to deal with negative numbers. The classes had been taught about negative numbers prior to the lessons on early algebra yet connecting the ideas was still a problem. This is a common problem in the teaching of early algebra, as the Researcher has found from many years of experience. The link between number and algebra, when it involves negative numbers, can be tenuous indeed. Understanding integers and the ability to use them fluently in calculations for learning algebra is important. The NZ Numeracy Project (Britt & Irwin, 2005) found that when students could represent number operational strategies successfully they were more likely to be able to generalize and therefore to illustrate algebraic thinking strategies.

#### **7.5 Comparison of the Teachers' Knowledge of, Beliefs and Practice concerning the Teaching of Algebraic Concepts**

All four teachers in the study found value in providing students with the means to build good foundations of knowledge, understanding and skills as referred to in the Revised Blooms Taxonomy (Krathwohl, 2002). Students who possess these foundational constructs can then develop their thinking and reasoning using algebra. Algebra is more than a continually developing body of knowledge as it is also a way of thinking, reasoning, talking and doing

(Kaput, 2008). Each of the four teachers worked to engage students and encourage interest in early algebra. This is evidenced by the strategies they used and the positive way they spoke of algebra in lessons.

Grace, Penny and Marie introduced algebraic notation, language and definitions in their lessons. Each modelled examples where algebraic words are used and each was explicit in their teaching of the definitions of these words. The majority of Penny's teaching was directed to imparting knowledge and understanding of the use of the language of algebra to her students. John did not focus noticeably on these as such during the lesson observations but did mention these incidentally as he worked with his class on finding and describing patterns. In interviews John explained the value he gave to language and that any algebraic terms he used had been defined prior to the lessons observed by the Researcher.

Grace and Marie had experience in teaching senior mathematics classes and this may have influenced the amount of emphasis they put on particular ideas. John, whilst being a middle school teacher, was aware of the content his students would require for the following year and his continuing collaboration with the Head of the Mathematics Department determined where his emphasis lay. Marie raised the issue of readiness more than once in interviews. Marie believes that teachers need to make sure that ideas are explored and information given when students are ready to learn them. It is important that the students have the prerequisite cognition to move forward in their thinking. Boaler (1999) proposed that if mathematics is seen as a subject where thinking is important then mathematics classrooms should be places where students are given many opportunities to develop their ability to think mathematically. Marie believed that a too early introduction to these ideas can be destructive to students' understanding. Kaput (2000) lamented that in some classrooms rules and procedures in early algebra take precedence over thinking and understanding. The Researcher found that Grace and Marie introduced students to more complex ideas in the four lessons.

## **7.6 Comparison of the Teachers' Knowledge of, Beliefs and Practice concerning the Connection of Established Number (Arithmetic) Concepts with the Effective Teaching and Learning of Early Algebra**

All four teachers in the study employed strategies that could be considered to come from all three of the orientations identified by Askew et al., (1997), namely, discovery, connectionist and transmission. These orientations Askew related to Ernest's suggestion that a teacher's belief system was made up of three components, namely:

- (a) the teacher's conception of the nature of mathematics as a subject for study;
- (b) the teacher's view of the nature of mathematics teaching; and
- (c) the teacher's view of the process of learning mathematics.

With Askew's model in mind, the Researcher chose to more closely analyse the observed strategies of the teachers. The Researcher categorized each observed strategy into the three orientations. When the Researcher observed a teacher working from a transmission orientation the strategies used put the teacher at the centre of the learning. The teacher was in control of the content and instructed the students. The teacher was responsible for communicating the language and conventions of early algebra. The teacher organized lesson content to meet the learning needs of the students. Process and procedures were demonstrated to students in a whole class lesson. The teacher would model thinking and working mathematically when working from a transmission orientation. Irwin and Britt (2007) propose that where the focus for arithmetic in the classroom curriculum is on developing an awareness of the underlying structure of the operations rather than on merely finding answers through the procedures and algorithms, it is likely there will be important positive consequences for the development of algebraic thinking. This conjecture is supported by the brain research reported by Sousa (2008).

When the Researcher observed a teacher working from a discovery orientation the strategies used put the student at the centre of the learning. The actions of the teacher in planning the learning opportunity for the students is crucial when the teacher's objective is for the student to discover an idea for themselves. Careful planning does not negate the fact that the students may well take the idea beyond what the teacher expected but at the very least

the main objective will be covered. Use of strategies from this orientation may well give the teacher more scope for differentiation of tasks for students. Researchers Herscovics and Linchevski (1994) proposed that it is the teacher's role to structure the learning environment and the learning opportunity for the student if the student is to bridge the 'cognitive gap'.

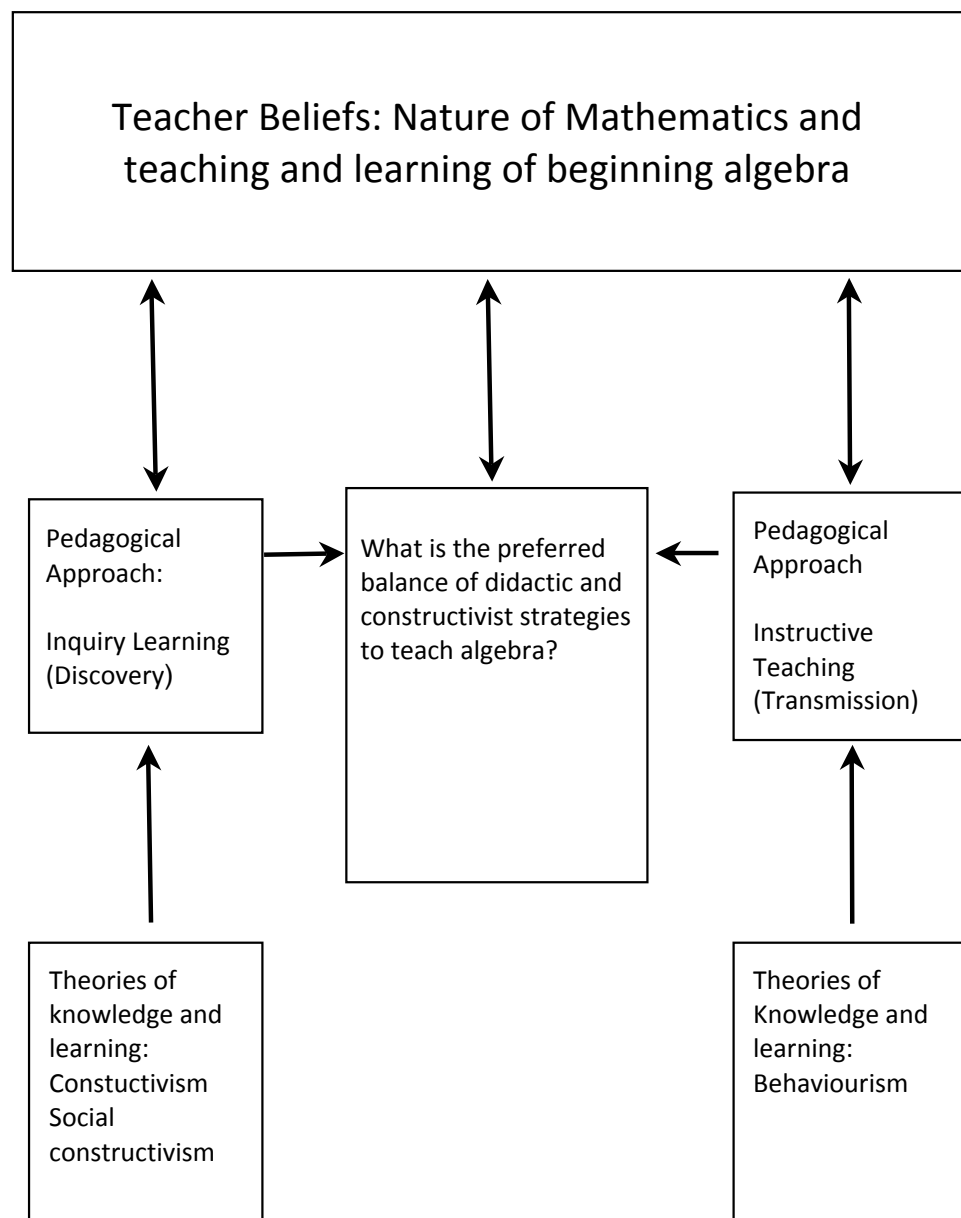
When the Researcher observed a teacher working from a connectionist orientation the strategies used had the teacher and the student at the centre of the learning in the classroom. The Researcher would suggest that connectionist strategies at heart are about thinking and working together in praxis. To make links between existing knowledge and new ideas, and to further develop students' ability to think in this way. The teacher has an important role to play in fostering student growth. It is in the meaningful exchanges between teacher and student that making connections between concepts and skills can make this happen.

Video segments from each of the four observed lessons for each teacher were transcribed and coded in the Artichoke program. Cluster analysis of the data of the four observed lessons taken by each teacher helped to further refine these ideas and the links between them. In each of the case studies, examples of teachers' actual choices of particular sets of strategies were seen to link logically, leading the Researcher to create four themes of practice emerging from the data. In the classroom observations, the teacher demonstrated one, two, or even three of the following themes of practice in their teaching:

- student focus;
- teacher focus;
- teaching/learning balance; and,
- teaching/learning and student balance.

The emergence of these four clear themes of practice from the data also necessitated a revision of the conceptual framework presented on the next page so as to better represent the Researcher's thinking about the findings of the study. The original conceptual framework is shown below. Teaching beliefs and confidence appeared to remain key determinants of what teachers did in the classroom. Therefore, in the revised conceptual framework, beliefs and confidence are still placed at the top of the figure, but they are now more closely related to relevant aspects of pedagogical content knowledge for the teaching and learning of early algebra. What did each teacher understand about how students learned about number and

algebra and the link between the two? The teachers' personal experience as teachers of early algebra must necessarily also have had an influence on their actions in the classroom, despite the fact that the exact nature of the relationship between beliefs and practice is as yet not totally clear to current Researchers (Buzeika, 1996).



*Figure 3 Conceptual Framework*

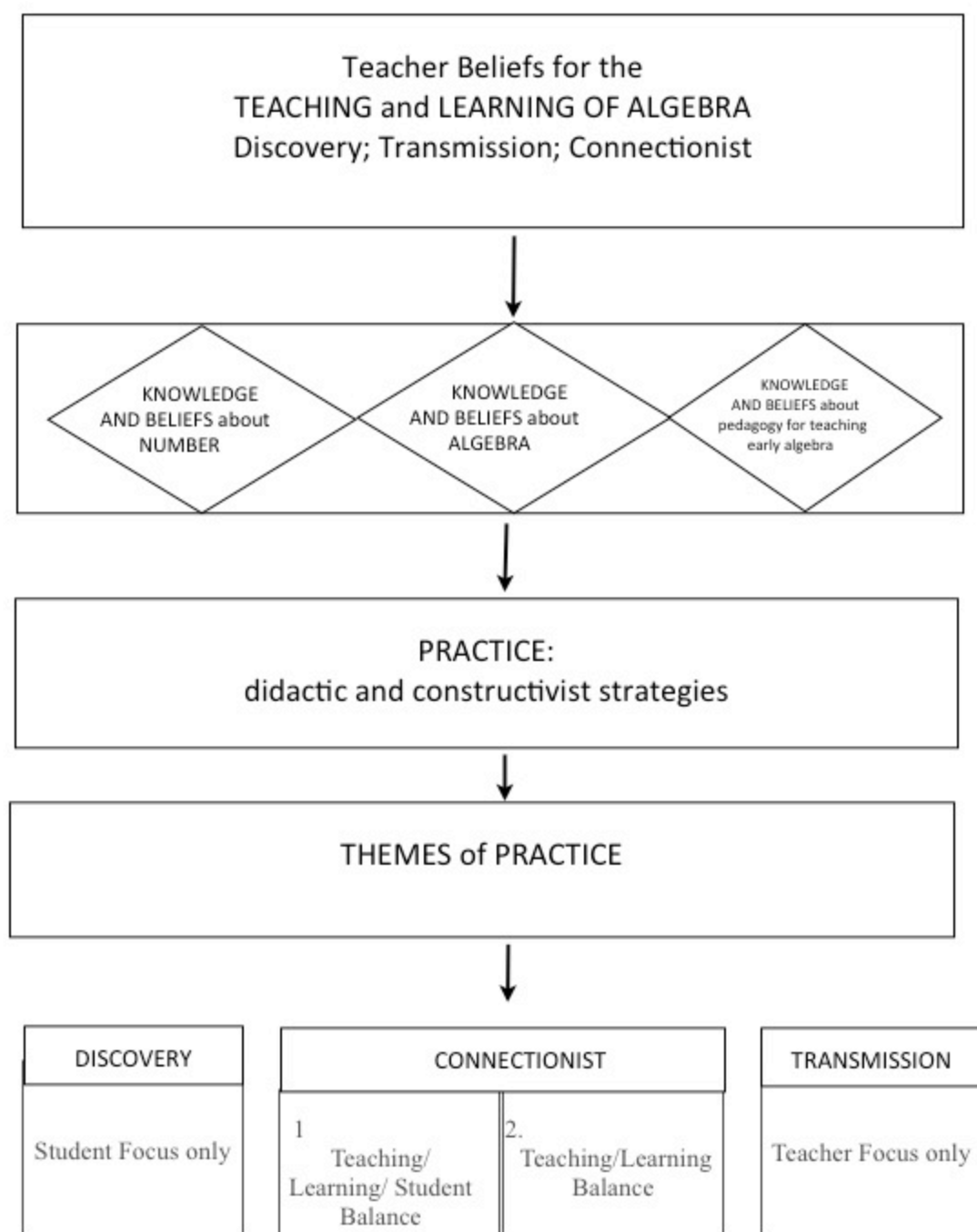


Figure 30 *Revised Conceptual Framework*

## 7.7 Summary Of Comparisons

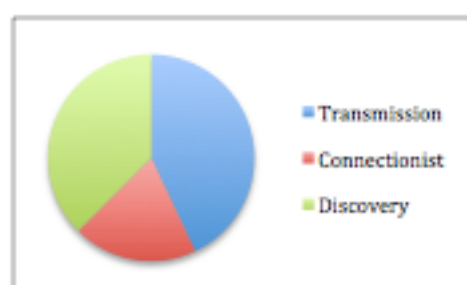
It is in the teachers' choice of strategies that theoretical underpinnings become transparent. To illustrate this the theme of student focus contained strategies the Researcher had identified as belonging to a discovery orientation. The theme of teacher focus contained strategies that the Researcher had identified from a transmission orientation. In a similar way in the themes of Teaching/learning balance and Teaching/learning/student balance, both themes contain strategies from the orientations of transmission, discovery and connectionist. These may at first seem similar, but the difference between the two is that the latter has more connectionist strategies than transmission or discovery. There is a balance to be found in these two themes of practice. This balance has at its centre connectionist strategies. The connectionist element is an important part both of the theme of practice Teaching/learning balance and Teaching/learning/student balance.

In his study Askew and his team of Researchers identified teachers possessing a 'strong orientation' that emerged from the data. In this study, the Researcher examined the predominance of each of the three orientations for each teacher. The teachers' beliefs were initially collected from Questionnaires I and II and these results were discussed in Chapter Four. The findings from the quantitative data are used to support the qualitative data collected in the study. The pie graphs in Figure 31 of this chapter provide the reader with a visual representation of each teacher's beliefs as expressed at the beginning of the study compared with their practice over the four lessons. As each teacher saw value in each of the orientations to teaching early algebra, and each used strategies identified coming from a particular orientation, the differences in their practice were reflected in the number of instances within a lesson that a particular bank of strategies was used. It must be noted that this study was not designed to measure best practice but to investigate teachers' practice in teaching early algebra.

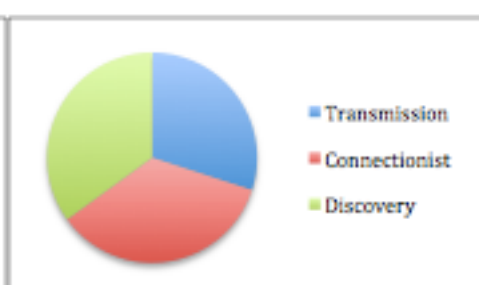
In Figure 31 the combined pie graphs show a comparison for each teacher between the responses teachers gave to the questionnaire on their beliefs about teaching and learning and their practice as recorded by the Researcher over four lessons in early algebra.



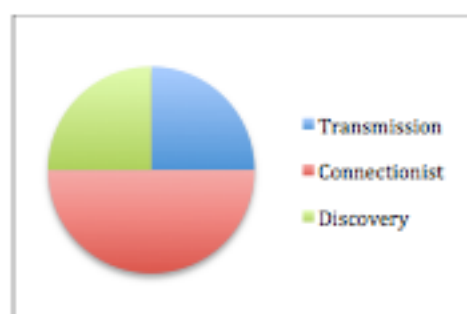
John's beliefs about teaching



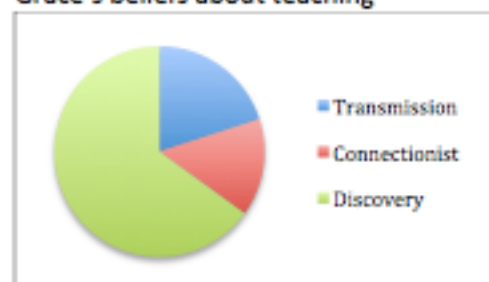
John's practice



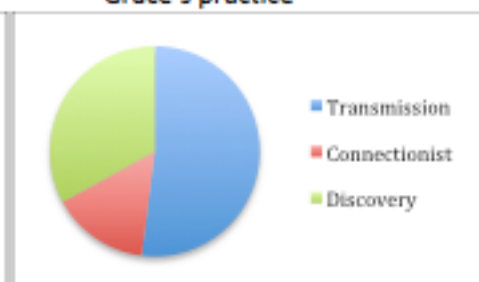
John's beliefs about learning



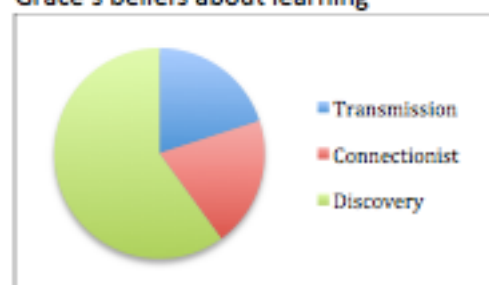
Grace's beliefs about teaching



Grace's practice

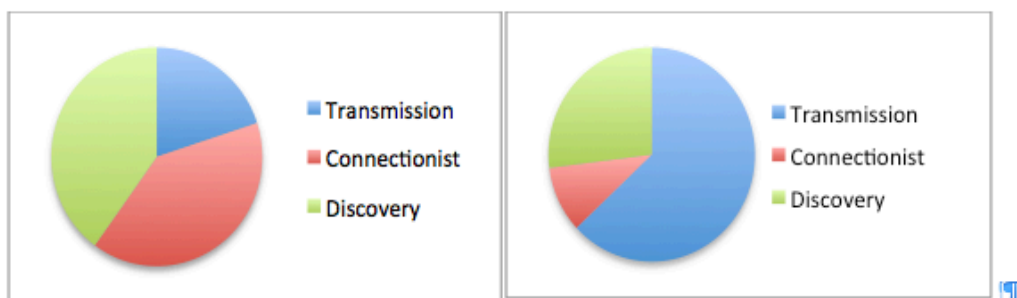


Grace's beliefs about learning

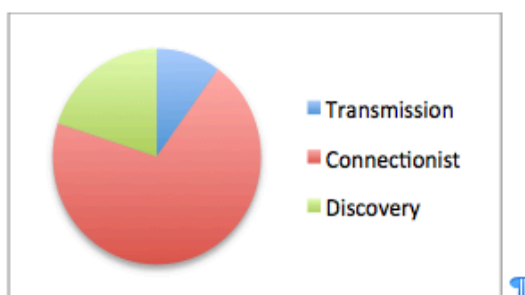


Penny's beliefs about teaching → →

→ Penny's practice ¶



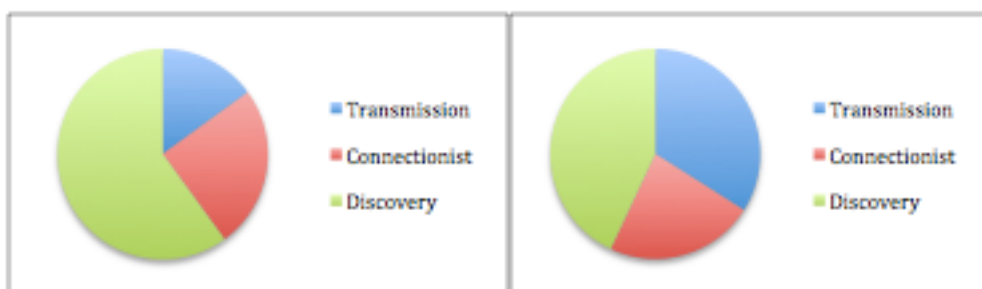
Penny's beliefs about learning ¶



¶

Marie's beliefs about teaching

Marie's practice



Marie's beliefs about learning

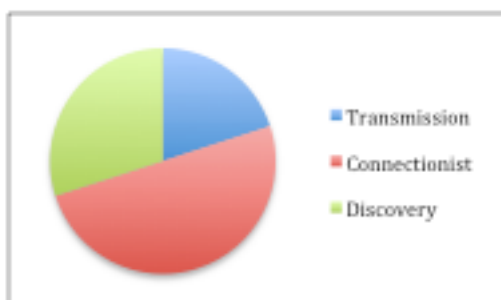
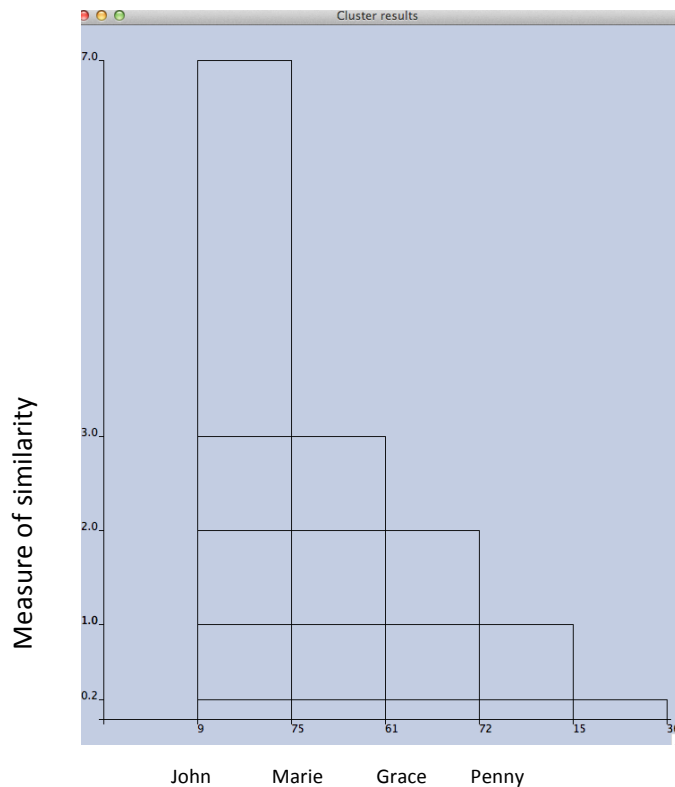


Figure 31 Comparison of teacher's beliefs and practices



*Figure 32 Cluster graph of teachers' practice*

In the graph in Figure 32, the numbers on the **y** axis measure similarities between the teachers. The larger the number the greater were the differences. The numbers on the **x** axis are the codes given to teachers.

(Please note: Code 36 denotes direct instruction and Code 15 denotes whole class instruction.)

Cluster analysis using Artichoke software (Fetherstone, 2012) showed that John varied in his practice most markedly from the other three teachers overall. This may be due to the fact that John trained as a primary teacher and therefore his background and experience would help determine how he approached the teaching of these early algebra ideas. However, it should also be noted that John did consult with his Head of Department often about what and how he was teaching. It may be that the differences in John's practice relate not only to his own preferences but also to the direction of practice advised by the mathematics department at the school. John's school makes extensive use of calculator technology in their teaching of algebra throughout the secondary school. John's beliefs about teaching and learning did not vary considerably from his practice. His response to the initial questionnaire

highlighted, the Researcher would suggest, a reciprocal belief about the practices of teaching and learning. John's response to Questionnaire I revealed a preference for a connectionist view of teaching and learning. However, in practice the strategies he used came from all three orientations, with almost equal weightings to transmission and discovery. The graph of John's practice in Figure 36 shows that his connectionist strategies are almost equally flanked by transmission and discovery. This is in common with Marie. The Researcher would assert that when students have been given the knowledge (transmission) and then through inquiry (discovery) have made this knowledge their own, dialogue (connection), understanding and generalization become possible. In early algebra when students are being initiated into the ideas and conventions it is reasonable to assume that the teacher as the "carrier of the tradition" (Sfard, 2000, p. 167) will explicitly teach these rules and the appropriate language. If students do not know these, expecting them to communicate algebraically can be doomed to failure.

John, in essence, interprets effective teaching in early algebra in a way which fits Askew's definition:

1. Valuing students' methods and teaching strategies with an emphasis on establishing connections within mathematics.
2. Valuing teaching as important and a view of mathematics as a collection of separate routines and procedures.
3. Valuing student learning as important and a view of mathematics as being discovered by students. (Adapted from the work of Askew et al., 1997)

Initially both Marie and Grace claimed to have a strong discovery orientation initially, yet this was not evident in their practice. Marie spoke about herself as a teacher who aimed to use more discovery strategies than any other but she reflected that this was something she aimed at but had not realized in her teaching. Grace also spoke of her use of open-ended tasks where students could discover for themselves the knowledge they would need to solve the problem she had set them. The Researcher would propose that it is both the nature of the subject matter, early algebra, and the students' learning needs which ultimately determined the strategies these experienced teachers chose to use.

Penny expressed the view that connectionist strategies should be a large part of her teaching, and even more so for her students' learning program, and yet her practice shows that she favoured those strategies identified as transmission. This disproportionate use of transmission strategies may be due to Penny's lack of experience or lack of knowledge of other strategies she could have used. Early algebra, when students transition in thinking from number to using letters, requires a good deal of creative thinking on the students' part. Discussion and problem-solving can assist the students to make sense of the need for algebra and how it can be used.

The issue for teachers is therefore not whether learners are capable, nor even whether learners will use those powers in lessons, but how to foster and support the use of those powers in mathematical ways, not only within mathematics but also so as to use mathematics to make sense of the world.  
(Mason et al., 2007, p.43)

At the focus group meeting it became clear to Penny that she needed to look for resources beyond her school. Penny also saw the value in seeking advice from more experienced teachers not in her school. Penny found her students' test results were disappointing. In the final interview with the Researcher, Penny expressed the view that perhaps her emphasis on transmission strategies had been to the detriment of student understanding and that it had become evident to her in the responses students made to the test questions that the understanding was not there for some students. The discussion held at the round table meeting provided Penny with ideas she intended to implement in her teaching.

The body of knowledge students need to know can be taught using transmission strategies; however, it is through the use of discovery strategies students can experience for themselves the thinking and ideas of early algebra and take ownership of their learning. It is the skilful teacher who takes both sets of strategies and uses a third set, namely those identified as connectionist. Connectionist strategies include:

1. helping students make connections to prior learning;
2. helping students make connections between ideas;
3. giving students thinking time for elaboration;

4. engaging students in the lessons; and,
5. encouraging students to make a significant contribution to lessons.

The inclusion of connectionist strategies complements transmission and discovery. (Askew et al., 1997). Students use what they have been told, shown or experienced to find for themselves. It makes sense and to knit these ideas together in order to grow as young mathematicians. Where the teachers in the study used a balance of strategies from the three orientations they found that in general their students achieved success in learning the early algebra they had covered.

Focus group discussions also had some impact on how the teachers subsequently thought about their lessons. When interviewing each teacher at the end of the data-gathering phase of the study the Researcher found that the notion of strategies based in the orientations of transmission, discovery or connectionism had led them to reflect on their lessons about their practice and to think about the balance of strategies they had used.

## **7.8 Chapter Summary**

In this chapter, the four case studies have been compared and contrasted to show differences and similarities. The analysis of the teachers' practice was founded on the three underlying teaching orientations of transmission, discovery and connectionist discussed throughout the thesis (Askew et al., 1997; Swan 2006).

All of the teachers in the study used more transmission or instructive strategies than their professed beliefs would have indicated. All teachers in the study did use a balance of teaching strategies, although there was some difference in the weightings that they gave to particular strategies within a lesson, and this appeared to be linked to their professed beliefs about the nature of algebra. Each of the teachers in the study demonstrated a positive view of the study of algebra to their students. The language that the teachers used and the strategies that they used to support and engage their students in the unit of work all contributed to a positive approach in the classroom. The greatest variation in content planning and teaching evident in the teachers' work was in the transitional material, namely moving students from working with number to algebraic variables. Only one of the four teachers in the study, Marie, explicitly taught the laws of number and then proceeded to link these to calculations using

variables. The other teachers appeared to assume that students would make the linkages themselves. The teachers did discuss operations with numbers in the context of early algebra lessons. However, the explicit teaching of the laws of number operations was not evident in three of the four cases.

A revised conceptual framework is provided with a summary of the comparisons made in the chapter and is shown in Figure 29. It is important to acknowledge the worldview of the Researcher who subjectively interpreted events in the light of personal experience as a teacher of many years' standing. From this worldview, interpretations of the findings were made, as were assertions. This cross case analysis focused on strengthening the answers to the research questions provided in Chapter Eight of the thesis.

In the chapter that follows the limitations of the study are acknowledged. The chapter concludes with implications for future research and teacher professional learning.

# Conclusions, Limitations and Implications

## 8.1 Introduction

This study investigated the practice of four teachers' as they supported their students transition from number to algebra. The research examined how teachers' beliefs (Ernest, 1989) and aspects of the teachers' pedagogical content knowledge (Shulman, 1987) underpinned the way they approached the teaching of algebra. The teaching orientations identified by Askew et al., (1997) were used to analyse both quantitative and qualitative data collected in this mixed methods study. The research questions that guided the study were thus:

1. What were the teachers' beliefs about the teaching and learning of algebra?
  - (a) How did they view the educational importance of algebra?
  - (b) How did they consider the teaching of number?
  - (c) How did they choose to balance transmission, connectionist and discovery strategies to teach beginning algebra?
2. How does reflection upon these particular beliefs and practices inform the teachers' methods for fostering the successful conceptual development of key beginning algebraic ideas?

A summary of the research findings is presented in this chapter as a set of conclusions. The limitations and implications are also considered.



## 8.2 Conclusions

### 8.2.1 Research question 1.

What were the teachers' beliefs about the teaching and learning of algebra?

(a) How did they view the educational importance of algebra?

Each of the teachers who volunteered to participate in this research project demonstrated, by their willingness to be involved, that they saw algebra as an important part of the mathematics curriculum.

In interviews all of the teachers talked about the importance of algebra. Each teacher took care to use positive language when introducing early algebraic concepts to students. In interviews and in classes the teachers' talked about the negative view that some parents and some people in the wider community can have about algebra and why this can happen. All of the teachers strove to explain the value to be gained from understanding and using algebra and that it was accessible to all. John in particular believes that learning algebraic concepts is achievable by all students. All of the teachers worked to engage students in their lessons in early algebra.

The teachers saw different aspects of algebra as being most important. Each of these aspects form a view of what algebra is and the importance of algebra in the Year Seven and Eight mathematics curriculum;

- algebra is about pattern
- algebra is useful for solving problems
- algebra is a way of thinking
- algebra has an aesthetic beauty.
- learning how to use algebra to solve problems or to describe relationships in Year Seven and Eight is good preparation for senior school mathematics.

(b) How did they consider the teaching of number?

Only one of the four teachers explicitly taught the laws of number as part of their work in early algebra. The teachers all agreed that good number skills and an understanding of number were important preparation for algebra. Working with negative and positive integers was given a transitory treatment by the teachers.

(c) How did they choose to balance instructional teaching with inquiry-based learning?

Two of the teachers believe that a discovery orientation results in best practice in teaching beginning algebra. Half of the teachers believe that a connectionist orientation best facilitates student learning. One of the teachers believe that both discovery and connectionist orientations best facilitate student learning and teaching. The teachers believe that their practice is approximately in the middle between student centred at one end of the spectrum and teacher centred at the other.

Over the four lessons, all of the teachers used more transmission strategies than discovery in their teaching (Table 21, p. 83). One teacher used more than double the amount of transmission strategies to discovery strategies. All teachers used some connectionist strategies but these were used much less frequently than those from the transmission or discovery strategies.

Of the four themes of practice classified by the Researcher, three of the teachers used strategies from the theme Teacher/learning/student balance to teach for the transition from number to algebra. This theme contains two connectionist strategies with one transmission and one discovery strategy.

The fourth teacher used strategies from the theme Teacher/learning balance to teach for the transmission from number to algebra. This theme contains one connectionist strategy with two transmission and two discovery strategies.

### 8.2.2 Research question 2

How does reflection upon these particular beliefs and practices inform the teachers' methods for fostering the successful conceptual development of key beginning algebraic ideas?

The teachers' beliefs underpinned their strategic choices, and were to some degree determined by aspects of their pedagogical content knowledge. In knowing how to teach algebra as a branch of mathematics the teachers brought different levels of experience to the classroom. This knowledge had a direct link to the emphases each teacher placed upon the concepts they taught, and how they planned their lessons. All of the teachers in the study used more transmission or instructive strategies than their professed beliefs would have predicted.

Analysis of the data gathered showed that the teachers' beliefs about what algebra is had a significant bearing on how they planned their teaching of these early algebraic concepts. It is noted that the four teachers in the study all adhered to the progression of the textbook or ebook they were working from with their class.

The example of John shows us firstly that teachers can and do practise in accordance with their stated beliefs about the nature of mathematics, teaching and learning. Secondly it shows also that an emphasis on pattern is critical to the learning and teaching of algebra, as is student discussion and collaboration.

The example of Grace shows us firstly that context, content and available resources may see a teacher practise contrary to their beliefs. Secondly it shows us that an understanding of number is central, and that practise further assists student learning. Finally, a balance of modelling of correct procedures and process with opportunities for students to discover ideas for themselves in early algebra can work together, if the teacher also employs strategies from a connectionist orientation.

The example of Penny shows us firstly that it is important for teachers to be supported if they are to improve their knowledge and understanding of best practice in teaching early algebra. Secondly, it is crucial for teachers to adopt a strategic approach to teaching from the number and algebra strand and to be mindful of the relationships within the strand. Thirdly, her example shows us that, given opportunities for professional learning, teachers can change

their views on practice for teaching and learning and develop a wider perspective of algebra itself.

The example of Marie shows us that employing a balance of strategies and a variety of resources in teaching early algebra can result in rich learning experiences for students. Also students' learn more powerfully when they discover concepts for themselves. Student engagement is important, and for this to occur the teacher needs to be cognizant of each student's background as a learner. Finally, teacher knowledge and understanding of how students learn early algebraic concepts, and an understanding of the progression of algebra in school, is crucial.

### **8.3 Limitations**

In this study the focus was on the teachers' beliefs and how these beliefs influenced the choice of strategies the teachers employed in the classroom when teaching students for the transition from number to algebra. There was no attempt made to compare the efficacy of particular practice in terms of student achievement, although the teachers themselves offered their assessment of their practice in terms of their students' results in the end of unit tests. No claims can therefore be made about the effectiveness of the pedagogical approaches reported in this study.

This was a small mixed methods study with only four teachers involved in the case. All of the teachers taught at schools that were very similar. They were low fee paying independent schools sharing a common ethos. In the NAPLAN testing in Numeracy the students average was band 7 for all four schools. Therefore the findings from this research cannot be generalized to other types of schools.

The period of observation was carried out over only four lessons and the teachers' planning documents were not examined so comparison of the teachers content is limited to the lesson observation phase of data gathering. However, there were similarities and differences to be explored. Availability issues limited the time open for visiting classrooms and therefore it was not possible to video four consecutive lessons for each teacher. The use of only one video camera also limited the data that could be collected using this method.

The schools the teachers worked in were low fee paying independent schools in Western Australia for students from Kindergarten to Year 12. The Year Seven and Eight students the teachers worked with belonged to the middle school within each school. In Western Australian schools there are several different models of middle years schooling. There are some schools where students as young as Year Six would be considered to be in the middle years of schooling. Traditionally, Year Seven has been the end of primary schooling and Year Eight the beginning of secondary schooling. In this study the focus was on teaching for the transition from number to algebra and the fact that the students were in the middle years of schooling was not an aspect explored. This was felt to be in the interests of maintaining the integrity of the research.

## **8.4 Implications**

### **8.4.1 Implications for practice**

Recent research investigating the relationship between theory and practice have also found that in teaching beginning algebra the teacher must examine their practice and reflect on their own understandings of algebra and the current curriculum (Agudelo-Valderrama et al., 2007; Malara & Vavarra, 2003; Zhang & Stephens, 2013). The Researcher was advised by a visiting tutor at a practicum that the need for detailed planning of lessons was essential to teach well, and approximately 50% of a teacher's workload involved planning. The findings of this research would support an emphasis on planning for student learning, not simply on the strength of the content to be taught.

Teachers need to consider the blend of strategies they use for teaching beginning algebra. What should be explicitly taught? What can be discovered working on a question? Where can the teacher help the student to connect new ideas to existing schema?

Findings from this study suggest that teachers should revisit number concepts in their lessons on a regular basis in beginning algebra, paying particular attention to negative integers. The teachers' raised the question of students' who come into Year Seven or Year Eight classes with relatively poor number skills in the focus group meeting. Research has shown the importance of students' having a sound understanding of number (Britt & Irwin, 2007). Not

merely in a computational sense but in the much wider sense of relationship (Blanton, et al., 2007). Finding time in an already busy curriculum can make it difficult for teachers in the secondary classroom to revisit number concepts as often as they would like. However, the Researcher would suggest that revisiting number concepts in a deep and meaningful way provides students the opportunity to transition to using algebra more seamlessly.

#### **8.4.2 Implications for research**

A possibility for future research would be to measure student learning and use this data to examine and compare the efficacy of a combination of strategies that the teacher used. To ask the question, what is the most effective blend of strategies teachers can use to teach beginning algebra?

#### **8.4.3 Implications for teacher professional learning**

This study has revealed benefits for teachers in reflecting on their practice. Opportunities should be provided for teachers to purposefully reflect on their practice, beliefs and knowledge about the teaching and learning of beginning algebra.

## REFERENCES LIST

- Agudelo-Valderrama, C., Clarke, B., & Bishop, A.J. (2007). Explanations of attitudes to change: Colombian mathematics teachers' conceptions of the crucial determinants of their teaching practices of beginning algebra. *Journal of Mathematics Teacher Education*. 10, 69-93  
Retrieved from <http://www.dx.doi.org/10.1007/s10857-007-9031-2>
- Alibali, M. W., & Nathan, M. J. (2007). Teachers' gestures as a means of scaffolding students' understanding: Evidence from an early algebra lesson. In R. Goldman, R. Pea, B. Barron, & S. J. Derry (Eds.), *Video research in the learning sciences* (pp. 349-365). Mahwah, NJ: Erlbaum.
- Askew, M., Brown, M., Rhodes, V., Johnson, D., & William, D. (1997). *Effective teachers of numeracy: Report of a study carried out for the Teacher Training Agency*. King's College London, U.K.
- Asquith, P., Stephens, A. C., Grandau, L., Knuth, E. J., & Alibali, M.W. (2005). *Investigating middle-school teachers' perceptions of algebraic thinking*. Montreal, Canada: American Educational Research Association.
- Australian Curriculum and Assessment Authority [ACARA]. (2009). Shape of the Australian curriculum mathematics framing paper. Retrieved from [www.acara.edu.au/verve/\\_resources/Australian\\_Curriculum\\_-\\_Maths.pdf](http://www.acara.edu.au/verve/_resources/Australian_Curriculum_-_Maths.pdf)
- Australian Curriculum and Assessment Authority [ACARA]. (2012). The Australian curriculum mathematics, F to Year 10. Retrieved from [www.australiancurriculum.edu/Mathematics/Curriculum/F-10](http://www.australiancurriculum.edu/Mathematics/Curriculum/F-10)
- Australian Curriculum and Assessment Authority [ACARA]. (2013). The Australian curriculum mathematics, F to Year 10. Retrieved from [www.australiancurriculum.edu/Mathematics/Curriculum/F-10](http://www.australiancurriculum.edu/Mathematics/Curriculum/F-10)
- Babbie, E. (2004). *Survey research methods* (2<sup>nd</sup>. Ed.). Belmont, CA: Wadsworth Publishing
- Bailey, D.H., & Borwein, J.M. (2012). Algebra is essential in a 21<sup>st</sup> century economy. *The Huffington Post*. Retrieved from [http://www.Huffingtonpost.com/david-bailey/algebra-is-essential-in-a\\_b\\_1724338](http://www.Huffingtonpost.com/david-bailey/algebra-is-essential-in-a_b_1724338)
- Ball, D. L. (1991). Teaching mathematics for understanding: What do teachers need to know about subject matter? In M. Kennedy (Ed.), *Teaching academic subjects to diverse learners* (pp. 63-83). New York: Teachers College Press.
- Ball, D. L. (2003). *Mathematical proficiency for all students: Toward a strategic research and development program in mathematics education*. Santa Monica, CA: RAND Corporation.  
[http://www.rand.org/pubs/monograph\\_reports/MR1643](http://www.rand.org/pubs/monograph_reports/MR1643).

- Ball, D. L., Hill, M.C., & Bass, H. (2005 ). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade and how do we decide? *American Educator*, (Fall) 14-46.
- Ball, D. L., Thames, M.H., & Phelps, H. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389-407.
- Hill, C., Rowan, B., Ball, D., & Schilling, S.G. (2008). Mathematical knowledge for teaching from unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, 39(4), p. 377
- Barron, B., & Engle, R.A. (2007). Analysing data derived from video records. In S. J. Derry (Ed.), *Guidelines for video research in education: Recommendations from an expert panel* (pp. 24-33). Retrieved from <http://drdc.uchicago.edu/what/video-research-guidelines.pdf>. (pp. 34-46)
- Battista, M. T. (1994). Teacher beliefs and the reform movement in mathematics education. *Phi Delta Kappan*, 75(6).
- Bishop, A. (1977). On loosening the contents. In L. Murray. (Ed.), *Meaningful mathematics*. Melbourne: Mathematics Association of Victoria.
- Black, T.R. (1999). *Doing quantitative research in social sciences: An integrated approach to research design, measurement and statistics*. Thousand Oaks, CA: Sage.
- Blanton, M. L., & Kaput, J. (2003). Developing elementary teachers' "algebra eyes and ears". *Teaching Children Mathematics*, 10(2), 70-77.
- Blanton, M., Schifter, D., Inge, V., Lofgren, P., Willis, C., Davis, F., & Confrey, J. (2007). Early algebra. In V.J. Katz (Ed.), *Algebra: Gateway to a technological future report* (pp.7-14). Washington, D.C.: The Mathematical Association of America.
- Boaler, J. (1998). A tale of two schools: Alternative teaching approaches and situated learning. In A. Watson (Ed.), *Situated cognition and the learning of mathematics* (pp. 83-93). University of Oxford: Oxford Centre for Mathematics Education Research.
- Boero, P., & Szendrei, J.R. (1998). Research and results in mathematics education: Some contradictory aspects. In J. Kilpatrick & A. Sierpiska (Eds.), *Mathematics as a research domain: A search for identity (An ICIM Study)* (pp. 197-212). Dordrecht: Kluwer.
- Borko, H., & Shavelson, R (1990). Teacher decision-making. In B.F. Jones & L. Idol (Eds.), *Dimensions of thinking and cognitive instruction* (pp.311-346). Elmhurst, IL: North Central Regional Educational Laboratory and Hillsdale, NJ: Erlbaum.
- Borko, H., Frykholm, J., Pittman, M., Eitejorg, E., Nelson, M., Jacobs, J., Koellner-Clark, K., & Schneider, C. (2005). Preparing teachers to foster algebraic thinking. *ZDM*, 37(1), 43-52.
- Brophy, J., & Good, T. (1986). Teacher behaviour and student achievement. In M. Wittrock (Ed.), *Handbook of research on teaching* (pp. 328-375). New York: Macmillan.



- Borrego, M., Douglas, E.P., & Amelink, C.T. (2009). Quantitative, qualitative, and mixed methods in engineering education. *Journal of Engineering Education*, 98(1), 53-66.
- Britt, M.S., & Irwin, K. C. (2008). Algebraic thinking with and without algebraic representation: A three-year longitudinal study. *ZDM Mathematics Education Educational Studies in Mathematics*, 40, 39-52.
- Brousseau, G. (2002). Theory of didactical situations in mathematics. In N. Balacheff, M. Cooper, R. Sutherland & V. Warfield (Eds.), *Didactique des mathematiques 1970-1990* Dordrecht, NL: Kluwer Academic Publishers.
- Burkhardt, H., Fraser, R., & Ridgeway, J. (1990). The dynamics of curriculum change. In I. Wirszup & R. Streit (Eds.), *Development in school mathematics education around the world (Vol. 2)* (pp 3-29). Reston, VA: National Council of Teachers of Mathematics [NCTM].
- Buzeika, A. (1996, Month). *Teachers' beliefs and practice: The chicken or the egg?* Paper presented at the Technology in Mathematics Education 19th Annual Conference of the Mathematics Education Group of Australasia (MERGA), Melbourne.
- Carpenter, T. P., Corbitt, M. K., Kepner, H. S., Lindquist, M. M., & Reys, R. E. (1980). Solving verbal problems: Results and implications from national assessment. *Arithmetic Teacher*, 28(1), 8-12.
- Charalambous, C.Y., Hill, H.C., & Mitchell, R. N. (2012). Two negatives don't always make a positive: Exploring how limitations in teacher knowledge and the curriculum contribute to instructional quality. *Journal of Curriculum Studies*, 44(4), 489-513.
- Cobb, P. (1994). Where is the mind? Constructivist and sociocultural perspectives on mathematical development. *Educational Studies in Mathematics*, 2, 105-109.
- Cobb, P. (1996). Where is the mind? A co-ordination of sociocultural and cognitive constructivist principles. In C. Fosnot (Ed.) *Constructivism: Theory, perspectives, and practice* (pp. 35-52). New York: Teachers College Press.
- Cobb, P., Wood, T., & Yackel, E. (1990). Classrooms as learning environments for teachers and researchers. *Journal for Research in Mathematics Education, Monograph (4)*, 125-146.
- Cobb, P., & McClain, K. (1999) Supporting teachers' learning in social and institutional context. In Fou-Lai Lin (Ed.), *Proceedings of the 1999 International Conference on Mathematics Teacher Education* (pp. 7-77). Taipei, Taiwan: National Taiwan Normal University.
- Cohen, L., Manion, L. (1994). *Research Methods in Education* (Fourth Ed.). New York: Routledge.
- Cronbach, L. J. (1951) Co-efficient alpha and the internal structure of tests. *Psychometrika*, 16(3), 297-334.
- Creswell, J. W., & Plano Clark, V.L. (2007). *Designing and conducting mixed methods research* (2<sup>nd</sup> Ed). Thousand Oaks California: Sage Publications Inc.

- Creswell, J.W., & Tashakkori, A. (2007) Editorial: Differing perspectives on mixed methods research. *Journal of Mixed Methods Research*, 1(4), 303-395  
[http://dx.doi.org/ 10.1177/1558689807306132](http://dx.doi.org/10.1177/1558689807306132)
- Creswell, J. W. (2008). *Educational research: planning, conducting, and evaluating quantitative and qualitative research*. New Jersey: Pearson Education Inc.
- Darling-Hammond, L. M., & McLaughlin, M.W. (2011). Policies that support professional development in an era of reform, *Phi Delta Kappan* 92 (6), 81-92. Retrieved from [www.kappanmagazine.org/cgi/content/full/92/6/81](http://www.kappanmagazine.org/cgi/content/full/92/6/81)
- Davis, E.B. (2005), A defence of mathematical pluralism. *Philosophia Mathematica*, 13(3), 252-276.
- Davis, R. B. (1975). Cognitive processes involved in solving simple algebraic equations. *Journal of Children's Mathematical Behaviour*, 1(3), 7-35.
- Dehaene, S. (2011). *The number sense: how the mind creates mathematics* (revised and updated edition). [Ebook library version]. Retrieved from <http://www.ebilib.com/>
- Devlin, K. J. (2000). *The math gene: How mathematical thinking evolved and why numbers are like gossip*. New York: Basic Books.
- Dick, B. (2005). *Grounded theory: a thumbnail sketch*. Retrieved from <http://www.aral.com.au/resources/grounded.html>
- Dockett, S. (1999). Thinking about play, playing about thinking. In Elizabeth Dau (Ed.). *Child's play: revisiting play in early childhood settings* (pp. 28-43). Rosebury NSW: MacLennan & Petty.
- Donaldson, M. (1978). *Children's minds*. Great Britain: Biddles Ltd.
- Engle, R. A., Conant, F. R., Greeno, J.G. (2007) Progressive refinement of hypothesis in video research. In R. Goldman, R. Pea, B. Barron, & S.J. Derry (Eds.), *Video research in the learning sciences* (pp. 239-254). Mahwah, NJ: Erlbaum.
- Erickson, F. (2006). Definition and analysis of data from videotape: Some research procedures and their rationales. In J. L. Green, G. Camilli, & P. B. Elmore (Eds.), *Handbook of complementary methods in education research* (pp. 177-205). Mahwah, NJ: Erlbaum.
- Ernest, P. (1991). *The philosophy of mathematics education*. London: Palmer Press.
- Ernest, P. (1998). *Social constructivism as a philosophy of mathematics*. Albany, NY: State University of New York Press.
- Ernest, P. (2004). Postmodernism and the subject of mathematics. In M. Walshaw & L. Burton (Eds.), *Mathematics education within the postmodern: The international perspectives on mathematics education series* (pp. 15-33). Connecticut: Information Age Publishing Inc.

- Ernest, P. (2010). Thinking intuitive understanding—big concepts. *Professional Educator*, 9(2), xx-xx.
- Ewing, B. (2011). Direct instruction In mathematics: Issues for schools with high indigenous enrolments: A literature review. *Australian Journal of Teacher Education*, 6(5).  
<http://dx.doi.org/10.14221/ajte.2011v36n5.5>
- Fennema, F., Franke, M.L. (1992). Teachers' knowledge and its impact. In D.G. Grouws (Ed.) *Handbook of research in mathematics teaching and learning* (pp. 147-164). New York: Macmillan.
- Fennema, F., Carpenter, T.P., Franke, M.L., Jacobs, V.R., & Empson, S.B. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education*, 27(4), 403-434.
- Fetherston, T. (2006). *Becoming an effective teacher*. Melbourne: Thomson.
- Filloy, E., & Rojano, T. (1989). Solving equations: The transition from arithmetic to algebra. *For the Learning of Mathematics*, 9(2), 19-24.
- Flores, M.M., & Kaylor, M. (2007). The effects of a direct instruction program on the fractions performance of middle school students at risk for failure in mathematics. *Journal of Instructional Psychology*, 34(2), 84-94.
- Fujii, T. (2003). Fostering an understanding of algebraic generalization through number expressions: The role of the quasi-variable. *12th ICMI Study Conference, The Future of the Teaching and Learning of Algebra*.
- Goldman, R., Pea, R., Barron, B. & Derry, S. J. (Eds.) (2007). *Video research in the learning sciences*. Mahwah, NJ: Erlbaum.
- Grossen, B. & Ewing, S. (1994). Raising mathematics problem-solving performance: Do the NCTM teaching standards help? Final report. *Effective School Practices*, 13(2), 79-91.
- Grinnel, R. (1988). *Social work research and evaluation*. Itasca, IL: F.E. Peacock.
- Guba, E., & Lincoln, Y. (2005). Paradigmatic controversies, contradictions and emerging confluences. In N. Denzin & Y. Lincoln (Eds.), *Handbook of qualitative research* (3<sup>rd</sup> Ed.) (pp. 191-215). Thousand Oaks, CA: Sage.
- Hallagan, J. E. (2006). The case of Bruce: A teacher's model of his students' algebraic thinking about equivalent expressions. *Mathematics Education Research Journal*, 18(1), 103-123.
- Handal, B. (2003). Teachers' mathematical beliefs: A review. *The Mathematics Educator*, 13(2), 47-57.
- Hargreaves, A. (1994). *Changing teachers, changing times: Teachers' work and culture in the post modern age*. New York: Teachers College Press.
- Hartley, J. (1998). *Learning and studying: A research perspective*. London: Routledge.
- Hattie, J. (2012). Visible learning for teachers [electronic resource] maximizing impact on learning. London; New York: Routledge.

- Herscovics, N., & Linchevski, L. (1994). A cognitive gap between arithmetic and algebra. *Educational Studies in Mathematics*, 27(1), 59-78.
- Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp.65-97). New York: Macmillan.
- Hill, C., Rowan, B., & Ball, D. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, 42(2), 371-406.
- Hill, C., Rowan, B., Ball, D., Schilling, S.G. (2008). Unpacking pedagogical content knowledge: conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, 39(4), 372-400.
- Irwin, K. C., & Britt, M.S. (2005). The algebraic nature of students' numerical manipulation in the New Zealand numeracy project. *Educational Studies in Mathematics*, 58(2), 169-188.
- Irwin, K. C., & Britt, M. S. (2007). The development of algebraic thinking: results of a three-year study. In B. Annan, F. Ell, J. Fisher, N. Hawera, J. Higgins, K.C. Irwin, ... J. Young-Loveridge (Eds.). *Findings from the New Zealand numeracy development projects 2006* (pp. 33-43), Wellington: Learning Media.
- Jaworski, B. (1994). *Investigating mathematics teaching: A constructivist enquiry*. London: The Falmer Press.
- Kaput, J. J. (1999). Teaching and learning a new algebra. In E. Fennema, & T. A. Romberg (Eds.), *Mathematics classrooms that promote understanding* (pp.133-155). Mahwah, NJ: Lawrence Erlbaum Associates.
- Kaput, J. J. (2000) Teaching and learning a new algebra with understanding. [Opinion Paper]. Dartmouth, M.A.: National Center for improving Student Achievement in Mathematics and Science.
- Kaput, J.J. (2008). What is algebra? What is algebraic reasoning? In J. Kaput, D. W. Carraher & M. L. Blanton (Eds.), *Algebra in the early grades* (pp. 390-419). New York: Macmillan.
- Kieran, C. (1981). Concepts associated with the equality symbol. *Educational Studies in Mathematics*, 12, 317-326.
- Kieran, C. (1988). Two different approaches among algebra learners. In A.F. Coxford (Ed.), *The ideas of algebra, K-12* (pp. 91-96). Reston,VA: National Council of Teachers of Mathematics.
- Kieran, C. (1992). The learning and teaching of school algebra. In D. A. Grouws *Handbook for research on mathematics teaching and learning* (pp. 390-419). New York: Macmillan.
- Kieran, C. (1991). Helping to make the transition to algebra. *Mathematics Teacher*, 84(3), 49-51.

- Kieran, C. (2001). Preparing teachers for handling students mathematical communication: Gathering knowledge and building tools. In F. L. Lin & T. J. Cooney (Eds.), *Making sense of mathematics teacher education*, (pp.187-205). Place: Publisher.
- Kieran, C. (2007). Learning and teaching algebra at the middle school through college levels. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 707-762). Charlotte, N: Information Age Publishing.
- Koellner, K., Jacobs, J., Borko, H., Schneider, C., Pitman, M., & Eiteljorg, E., et al. (in press). The problem-solving cycle: A model to support the development of teachers' professional knowledge. *Mathematical Thinking and Learning*.
- Kuhn, T. (1970). Postscript 1969. In T. Kuhn *The structure of scientific revolutions* (2nd Ed.) (pp. 174-210). Chicago: University of Chicago Press.
- Kilpatrick, J. (1992). A history of research in mathematics education. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning*. New York: Macmillan.
- Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). In *adding it up: Helping children learn mathematics*. Mathematics Learning Study Committee, National Research Council. Retrieved from [www.nap.edu/catalog/9822.html](http://www.nap.edu/catalog/9822.html)
- Kilpatrick, J. (2010). Theories of mathematics education: seeking new frontiers. *The International Journal of Mathematics*, 42(5), Preface.
- Knuth, E. J., Stephens, A. C., McNeil, N. M., & Alibali, M. W. (2006). Does understanding the equal sign matter? *Evidence from solving equations. Journal for Research in Mathematics Education*, 37(4), 297-311.
- Koedinger, K. R., & Nathan, M. J. (2000). An investigation of teachers' beliefs of students' algebra development. *Cognition and Instruction* 18(2): 209-237.
- Koehler, M. S., & Grouws, D. A. (1992). Mathematics teaching practices and their effects. In D. Grouws (Ed.), *Handbook of research in mathematics teaching and learning* (pp.115-126). New York: Macmillan.
- Krathwohl, D. R. (2002). A revision of bloom's taxonomy: An overview. *Theory into Practice*, 41(4), 212-218.
- Kumar, R. (2011). *Research methodology a step-by step guide for beginners*. London: Sage Publications Ltd.
- Kyeleve, J. I., & Williams, J. S. (1996). Measures of teachers' attitudes towards mathematical modelling. *Proceedings of the 20th Conference of the International Group for the Psychology of Mathematics Education*, Valencia, Spain: Universitat de Valencia.

- Kyeleve, J. I., & Williams, J. S. (2006). Evaluating teachers' attitudes to curriculum reform: Teachers' beliefs in the importance of and perceived practical of mathematical modelling scale. *Journal of Education*, 11, 1-13.
- Lamb, J., Cooper, T., & Warren, E. (2007). Combining teaching experiments and professional learning: Conflicts between research and teacher outcomes. *Mathematics Education Research Journal*, 19(3), 73-92.
- Leedy, P.D., & Ormrod, J.E. (2001). Practical research: Planning and design (7<sup>th</sup> Ed.). Upper Saddle River, NJ: Merrill.
- Linchevski, L., & Sfard, A. (1994). The gains and pitfalls of reification: the case of algebra. *Educational Studies in Mathematics*, 26, 191-228.
- Lincoln, Y.S., Guba, E.G. (1985) *Naturalistic inquiry*. Beverly Hills, CA: Sage Publications.
- Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. New Jersey: Erlbaum Associates.
- Maat, S.M., Zakaria, E., Nordin, N.M., & Meerah, T.S.M. (2011). Confirmatory factor analysis of the mathematics teachers' teaching practices instrument. *World Applied Sciences Journal*, 12(11), 2092-2096.
- MacGregor, M., & Stacey, K. (1993). Cognitive models underlying students' formulation of simple linear equations. *Journal for Research in Mathematics Education*, 24, 217-232.
- MacGregor, M., & Stacey, K. (1999). A flying start to algebra. *Teaching Children Mathematics*, 6(2), 217-232.
- Mason, J. D., & H Bills, L. (2007). Studies in the zone of proximal awareness. Proceedings of the 30<sup>th</sup> annual conference of the Mathematics Education Research Group of Australasia: MERGA, Place
- McCrory, R., Floden, R., Ferrini-Mundy, J., Reckase, M.D., & Senk, S.L. (2012). Knowledge of algebra for teaching: A framework of knowledge and practices. *Journal for Research in Mathematics Education*, 43 (5), 584-615.
- McNeil, N. G., & Alibali, M. W. (2005). Knowledge change as a function of mathematics experience: All contexts are not created equal. *Journal of Cognition and Development*, 6(2), 285-306.
- McNeil, N., Grandau, L., Knuth, E., Alibali, W., Stephens, A., Hattikudur, S., & Krill, D. (2006). Middle-school students' understanding of the equal sign: The books they read can't help. *Cognition and Instruction*, 24(3), 367-384.
- Miles, M.B., & Huberman, A.M. (1994). *Quantitative data analysis: An expanded sourcebook* (2<sup>nd</sup> ed.). Thousand Oaks, CA: Sage.
- Miller, K. F., & Zhou, X. (2007). Learning from classroom video: What makes it compelling and what makes it hard. In R. Goldman, R. Pea, B. Barron, & S. J. Derry (Eds.), *Video research in the*

- learning sciences* (pp. 321-334). Mahwah, NJ: Erlbaum.
- Mischler, E. G. (1991). Representing discourse: The rhetoric of transcription. *Journal of Narrative and Life History*, 1, 255-280.
- Mulligan, J., Cavanagh, M., & Keanan-Brown, D. (2012). The role of algebra and early algebraic reasoning in the Australian curriculum: Mathematics. In B. Atweh, M. Goos, R. R. Jorgensen & D. Siemon, (Eds.). *Engaging the Australian national curriculum: Mathematics-perspectives from the field*. Online Publication: Mathematics Education Research Group of Australasia pp.47-70.
- Nathan, M. J., Kintsch, W., & Young, E. (1992). A theory of algebra word problem comprehension and its implications for the design of computer learning environments. *Cognition and Instruction*, 9(4). 329-389.
- Nathan, M. J., Koedinger, K. R., & Tabachneck, H.J.M. (1994). Toward a theoretical account of strategy use and sense-making in mathematics problem solving. *Proceedings of the 16th Annual Conference of the Cognitive Science Society* (pp.836-841). Hillsdale: Erlbaum.
- Nathan, M. J., Koedinger, K. R., & Tabachneck, H.J.M. (1996). Difficulty factors in arithmetic and algebra: the disparity of teachers' beliefs and student performance. *Proceedings of the American Educational Research Association[AERA] annual meeting*. New York: Publisher. .
- Nathan, M.J., & Koedinger, K.R. (2000). An investigation of teachers' beliefs of students' algebra development. *Cognition and Instruction*, 18(2), 209-237.
- National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: NCTM.
- National Council of Teachers of Mathematics. (1995). *Assessment standards for school mathematics*. Reston, VA: NCTM.
- National Council of Teachers of Mathematics. (2008). Algebra: what, when and for whom  
<http://www.ntcm.org/about/content.aspx?id=16229>
- Neuman, W.L. (2006). *Social research methods: Qualitative and quantitative approaches*. Boston: Pearson A and B.
- Neyland, J. (1995). Eight approaches to teaching mathematics. In J Neyland (Ed.), *Mathematics Education: A handbook for teachers Vol. 2* (pp. 34-48). Wellington, NZ: The Wellington College of Education.
- Northfield, J., Gunstone, R., & Erickson, G. (1996). A constructivist perspective on science teacher education. In R Treagust, R. Druit & B. Fraser (Eds.), *Improving teaching and learning in science and mathematics* (pp. 201-211). New York: Teachers College Press.
- Novak, J. D. (1992). Clarify with concept maps. *The Science Teacher*, 58(7), 45-48.

- Ormond, C. (2013). Selected document from workshop materials for ECU pre-service teacher Education Unit, "Transition mathematics: Early algebraic thinking for Years 5 to 8." Perth, WA: Edith Cowan University.
- Owen, J., Johnson, N., Clarke, D.M., Lovitt, C., & Morony, W. (1988). *Guidelines for consultants and curriculum leaders*. Melbourne: Curriculum Corporation.
- Pajares, F. M. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Preview of Educational Research*, 62(3), 307-331.
- Patton, M. Q. (1990). *Qualitative evaluation and research methods* (2<sup>nd</sup> Ed.). Beverly Hills, CA: Sage.
- Pea, R. D., & Hay, K. (2003). Report to the National Science Foundation: CILT workshop on digital video inquiry in learning and education, November 25-26, 2002 (based on NSF #0124012). Stanford, CA: Stanford Center for Innovations in Learning.
- Pea, R., & Hoffert, E. (2007). Video workflow in the learning sciences: Prospects of emerging technologies for augmenting work practices. In R. Goldman, R. Pea, B. Barron, & S. J. Derry (Eds.), *Video research in the learning sciences* (pp. 427-460). Mahwah, NJ: Erlbaum.
- Pea, R., Lemke, J. (2007) Sharing and reporting video work. In S.J. Derry (Ed.) *Guidelines for Video Research in Education. Recommendations from an Expert Panel* (p. 33). Retrieved from <http://drdc.uchicago.edu/what/video-research-guidelines.pdf>
- Perry, B., Howard, P., Tracey, D. (1999). Head mathematics teachers' beliefs about the learning and teaching of mathematics. *Mathematics Education Research Journal*, 11(1), 39-53.
- Perso, T. F. (1993). *Algebra outcomes for your class K-9*. Perth: Mathematical Association of Western Australia.
- Perso, T. F. (2004). Teaching mathematics in the middle years: Is middle school necessary? *Australian Mathematics Teacher*, 61(1), 25-31.
- Phillips, D.C., Soltis, J.F. (2004). *Perspectives on Learning: Thinking about Education series* (4<sup>th</sup> Ed.). New York, NY: Teachers College Press.
- Piaget, J. (1952). *The child's conception of number*. London: Routledge and Kegan Paul.
- Programme for International Student Assessment [PISA], 2003. Mathematics teaching and learning strategies: Programme for International Student Assessment. Retrieved from: [www.oecd.org](http://www.oecd.org)
- Polya, G. (1971, 1957). *How to solve it: A new aspect of mathematical method*. Princeton, NJ: Princeton University Press.
- Polya, G. (1965). *Mathematical discovery volume 2*. New York: Wiley & Sons Ltd.
- Punch, K.F. (1998). *Introduction to social research: Quantitative and qualitative approaches*. London: Sage.



- Radford, L. (2001). Signs and meanings in students' emergent algebraic thinking: A semiotic analysis. *Educational Studies in Mathematics*, 42, 237-268.
- Resek, D. D., and Kirshner, D. (2000). Interference of instrumental instruction in subsequent relational learning. *Journal for Research in Mathematics Education*, 31(5)
- Romberg, T. A. (1994). Classroom instruction that fosters mathematical thinking and problem solving: Connections between theory and practice. In A. H. Schoenfeld (Ed.), *Mathematical thinking and problem solving* (pp. 287-304). Hillsdale, NJ: Erlbaum.
- Schoenfeld, A. H. Creating an appropriate algebra experience for all grades K-12 students, Report of working group 1. In C. B. Lacampagne, W. Blair & J. Kaput (Eds.), *The Algebra Initiative Colloquium Volume 2 Papers presented at a conference on reform in algebra* (pp. 11-18). U.S. Department of Education: Retrieved from [www.files.eric.ed.gov/fulltext/ED385437](http://www.files.eric.ed.gov/fulltext/ED385437)
- Schoenfeld, A. H. (2004). *The math wars educational policy*, 18, 253-286.
- Sfard, A. (1991). On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin. *Educational Studies in Mathematics*, 22, 1-36.
- Sfard, A., Linchevski, L (1994). The gains and pitfalls of reification: The case of algebra. *Educational Studies in Mathematics*, 26(2/3), Learning Mathematics: Constructivist and Interactionist Theories of Mathematical Development (pp. 191-228). Retrieved from [www.jstor.org/stable/3482784](http://www.jstor.org/stable/3482784)
- Sfard (2000b). On reform movement and the limits of mathematical discourse. *Mathematical Thinking and Learning*, 2(3), 157-169.
- Stacey, K., & MacGregor, M. (1997). Ideas about symbolism that students bring to Algebra. *The Mathematics Teacher*, 90(2), 110-113.
- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15 (4), 4-14.
- Siegler, R. S. (1998). *Children's thinking* (3<sup>rd</sup> Ed.). Eaglewood Cliffs, N.J: Prentice Hall.
- Silverman, J., & Thompson, P. W. (2008). Toward a framework for the development of mathematical knowledge for teaching. *Journal of Mathematics Teacher Education*, 11, 499-511. Retrieved from <http://patthompson.net/PDFversions/2008SilvermanThompsonMKT.pdf>.
- Simpson, J., & Weiner, E. (Eds.), (1989). *The Oxford English Dictionary* (2nd. Ed.) Oxford: Oxford University Press. XVII.
- Sirotnik, K. A. (1999). Making sense of educational transformation. *Phi Delta Kappan*, 80(8), 606-610.
- Skemp, R. R. (1978). Relational understanding and instrumental understanding. *Arithmetic Teacher*, 26 (3), 9-15.

- Skemp, R.R. (1987). *The psychology of learning mathematics*. Hillsdale NJ: Lawrence Erlbaum Associates, Inc.
- Sousa, D.A. (2008). *How the brain learns mathematics*. Thousand Oaks, CA: Corwin Press.
- Sozniak, L., Ethington, C. A., & Varelas, M. (1991). Teaching mathematics without a coherent point of view: Findings from the IEA Second International Mathematics Study. *Journal of Curriculum Studies*, 23, 119-131.
- SPSS Statistics for Mackintosh (2012) version 21.
- Sriraman, B. E., & Lyn D. (2009). *Theories of mathematics education*: Springer.
- Staub, F.C., & Stern, E. (2002). The nature of teachers' pedagogical content beliefs matters for students' achievement gains: Quasi-experimental evidence from elementary mathematics. *Journal of Educational Psychology*, 94(2), 344-355.
- Steffe, L. (2001). A new hypothesis concerning children's fractional knowledge. *Journal of Mathematical Behaviour*, 20(3), 267-307.
- Swan, M. (2005). Designing and using research instruments to describe the beliefs and practices of mathematics teachers. *Research in Education*, 75, 58-70.
- Tabachneck, H. J. M., Koedinger, K. R., & Nathan, M. J. (1995). A cognitive analysis of the task demands of early algebra. *The Seventeenth Annual Conference of the Cognitive Science Society*, Hillsdale, N.J: Erlbaum.
- Tashakkori, A., & Teddie, C. (Eds.), (2003). *Handbook of mixed methods in social and behavioural research*. Thousand oaks, CA: Sage Publications.
- Taylor, H., & Taylor, L. (1993). *George Polya: Master of discovery*. Palo Alto, CA: Dale Seymour Publications.
- Teaching and Learning International Survey [TALIS]. (2009). Creating effective teaching and learning environments: First results from TALIS. Retrieved from [www.oecd.org/edu/talis/firstresults](http://www.oecd.org/edu/talis/firstresults).
- Telese, T. A. (1999). The role of social constructivist philosophy in the teaching of school algebra and in the preparation of mathematics teachers. Eric Document Reproduction Service (ED632469) PP.1-15.
- Thomas, R.M. (2003). *Blending qualitative and quantitative research methods in theses and dissertations*. Thousand Oaks, CA: Corwin Press, Inc.
- Thomson, S., Hillman, K., & Wernert, N. (2012). Trends in international mathematics and science study Monitoring Australian-Year-8-student-achievement internationally: TIMSS 2011, Victoria: ACER
- Torner, G. (2002). Mathematical beliefs. In G. C. Leder E. Pehkonen & G. Torner. (Eds.), *A Hidden Variable in Mathematics Education?* Dordrecht: Kluwer Academic Publishers.
- Turner, F. & Rowland, T. (2008) The knowledge quartet: a means of developing and deepening

- mathematical knowledge in teaching, *Mathematics Knowledge in Teaching Seminar Series: Developing and deepening mathematical knowledge in teaching (Seminar 5)*, Loughborough University, Loughborough.
- Turner, F. (2012). Using the knowledge quartet to develop mathematics content knowledge: The role of reflection in professional development. *Research in Mathematics Education*, 14(3), 253-271.
- Van Dooren, W., Verschaffel, L., & Onghena, P. (2002). The impact of preservice teachers' content knowledge on their evaluation of students' strategies for solving arithmetic and algebra word problems. *Journal for Research in Mathematics Education*, 33(5), 319-351.
- Verzoni, K., & Koedinger, K. R. (1997). Student learning of negative number: A classroom study and difficulty factor assessment. *Paper presented at the annual meeting of the American Educational Research Association*: Chicago.
- Von-Glaserfeld, E. (1990). An exposition of constructivism: Why some like it radical. *Monographs of the Journal from Research in Mathematics Education*, 4 (pp. 19-29). Reston, VA: National Council of Teachers of Mathematics.
- Vygotsky, L. S. (1978). In M. Cole, V. John-Steiner, S. Scribner, & E. Souberman, (Eds.), *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.
- Warren, E. (2003). The role of arithmetic structure in the transition from arithmetic to algebra. *Mathematics Education Research Journal*. 15(2). 122-137.
- Warren, E., Cooper, T., & Lamb, J. (2006). Teacher professional development in patterns and algebra: being sensitive to a teachers' zone of proximal development. Identities, cultures and learning spaces: Mathematics Education Research Group Australasia.
- Wellington, J. (2000). *Educational research: Contemporary issues and practical approaches*. London: Continuum.
- Wilson, M. S., & Cooney, T. (2002). Mathematics teacher change and development: The role of beliefs. In E. P. G. Leder & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education* (pp. 127-148). Dordrecht: Kluwer.
- Woodbury, S. (2000). Teaching towards the big ideas of algebra. *Mathematics Teaching in the Middle School*. 6(4) 226-301.
- Wu, H. (2001). How to prepare students for algebra. *American Educator/American Federation of Teachers*, 25(2), 8.
- Yates, L. (2004). What does good education research look like? In H. Torrance (Series Ed.),
- Yin, R. K. (2009). Case study research design and methods. Thousand Oaks: SAGE Inc.
- Yin, R. K. (2012). Applications of case study research. Thousand Oaks: SAGE Inc

Zang, Q., & Stephens, M. (2013). Utilising a construct of teacher capacity to examine national curriculum reform in mathematics. *Mathematics Education Research Journal*, 25(4), 481-502.

# APPENDICES

## Appendix I: Teacher Questionnaire I on beliefs.

Component/

Characteristic	Statement	Percentage
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### Section 1

#### *Mathematics is:*

MT	A body of knowledge and standard procedures a set of universal truths and rules, which need to be conveyed to students	<input type="text"/>
MD	A creative subject in which the teacher should take a facilitating role, allowing students to create their own concepts and methods	<input type="text"/>
MC	An interconnected body of ideas which the teacher and the student create together through discussion	<input type="text"/>

### Section 2

#### *Learning is:*

LT	An individual activity based on watching, listening and imitating until fluency is attained	<input type="text"/>
LD	An individual activity based on practical exploration and reflection	<input type="text"/>
LC	An inter personal activity in which students are challenged and arrive at understanding through discussion	<input type="text"/>

### Section 3

#### *Teaching is:*

TT	Structuring a linear curriculum for the students; giving verbal explanations and checking that these have been understood through practice questions; correcting misunderstandings when students fail to 'grasp' what is taught	<input type="text"/>
TD	Assessing when a student is ready to learn; providing a Stimulating environment to facilitate exploration; avoiding Misunderstandings by the careful sequencing of experiences	<input type="text"/>
TC	A non-linear dialogue between teacher and student in which Meanings and connections are explored verbally. Misunderstandings are made explicit and worked on	<input type="text"/>

Key to first column: The first letter represents Mathematics(M), Learning(L) or (T) Teaching. The second letter refers to Transmission (T), Discovery(D) or Connectionist (C) beliefs

**Appendix 2: Teacher questionnaire II on practice related to beliefs.**

(5) almost always, (4) most of the time, (3) half the time, (2) occasionally, (1) almost never.

Question		5	4	3	2	1
1	Students learn through doing exercises					
2	Student's work on their own, consulting a neighbour from time to time					
3	Students use only the methods I teach them					
4	Students start with easy questions and work up to the harder questions					
5	Students choose which questions they tackle					
6	I encourage students to work more slowly					
7	Students compare different methods for doing questions					
8	I teach each topic from the beginning, assuming they know nothing					
9	I teach the whole class at once					
10	I try to cover everything in a topic					
11	I draw links between topics and move back and forth between topics					
12	I am surprised by the ideas that come up in a lesson					
13	I avoid students making mistakes by explaining things carefully first					
14	I tend to follow the text or the workbooks closely					
15	Students learn through discussing their ideas					
16	Students work collaboratively in pairs or small groups					
17	Students invent their own methods					
18	I know exactly what maths the lesson will contain					
19	I tell students which questions to tackle					
20	I encourage students to make and discuss mistakes					
21	I only go through one method for doing each question					
22	I find out which parts students already understand and do not teach those parts					
23	I teach each student differently according to individual needs					

24	I jump between topics as the need arises					
25	I tend to teach each topic separately					

### **Appendix 3: Interview questions**

#### **{Set One}**

1. Approximately many years have you been teaching?
2. Of this time how much of it has been spent teaching algebra?
3. What is your experience of teaching algebra to year seven students?
4. How do you feel about teaching algebra?
5. Would you say that algebra is an important strand of mathematics? please give reasons for your answer?
6. Are there any particular hurdles, in the study of algebra, which you have observed students experience in the early work?
7. In your opinion what would be the major causes of problems in learning algebraic strategies and techniques?

#### **{Set Two}**

1. Did you find what you expected to find in terms of the outcomes of student learning of algebra at the end of the unit?
2. If you found any changes, what were these and can you suggest why this may have been the case?
3. At the completion of the unit of work on algebra how would you describe your level of confidence in teaching this subject matter please give reasons for your answer?



## **Appendix 4: Focus Group Meeting**

Teachers' 1, 2, 3 and 4 met together with the researcher to examine a unit of work on early algebra. (Appendix 9). There were 18 lessons, which would equate in some schools to a full terms program. The unit of work planned by the researcher used the national curriculum in the planning of the document.

A short power point presentation (Appendix 5) was made by the researcher, which provided teachers with guidelines to assist identification of teaching strategies and the theory that underpins them.

### **PART 1**

The four teachers' in the study were asked to identify the major practice (weight of strategies) employed by the teacher in each lesson. Then they had discuss the rational for their choices with each other.

How does the order of the material presented to students agree or differ from the Teachers' experience?

What difficulties or problems arising from the proposed unit of can teachers' predict if any?

### **PART 2**

The four Year 7 teachers were asked to read and comment on three detailed lessons from the unit of work on early algebra (Appendix 9).

For each lesson the teachers were asked to answer the following questions and to discuss with the group their reasoning.

Discussion questions

How does this detailed lesson plan confirm or change your first impression of the predominant teaching strategies to be employed?

How would you describe your previous experience of using each of the teaching strategies outlined in the lesson?

## **Appendix 5: Focus Group Meeting Power point Slides**

### Slide 1

Research questions- Original

- What are teachers' beliefs about the teaching and learning of algebra?  
Explicit teaching and inquiry based learning  
  
Value and importance of algebra  
  
Self efficacy beliefs for teaching algebra
- How do these beliefs inform current classroom practices used by teachers' to foster conceptual development of key beginning algebraic ideas?

### Slide 2

Teaching from Transmission Orientation (Askew et al. 1997)

- Mathematics is a series of rules and truths
- Teaching is of the "talk and chalk model".
- Students are expected to practice individually until fluency is achieved.

### Slide 3

Teaching from a Discovery Orientation (Askew et al. 1997)

- Mathematics is a human creation
- The teacher adopts a facilitator role
- Students make meaning for themselves through individual exploration and reflection.
- They learn often by trial and error to build on their existing knowledge.
- The student is at the centre of the learning.

Slide 4

**Balance**

What is the preferred balance of didactic and constructivist strategies to teach algebra?



Slide 5

Teaching from a connectionist orientation (Askew 1997 et al)

- The teacher views mathematics as a network of ideas
- The teacher must construct this network together with the student through collaborative discussion.

## **Appendix 6: List of strategies**

Allow students to investigate

Allow students to make errors

Ask students for reasoning

Closed questions

Definitions presented explained

Direct instructions

Follow textbook-worksheets

Calculator use

Identified content at the start of the lesson

Identify likely errors or confusion

Lesson starter

Manipulatives issued to students

Model procedure or process

Notes provided by teacher

Choice of questions

Self assessment or peer assessment

Students make their own notes

Students work individually

Students work with others

Teacher assessment

Use of practice in class

Whole class instruction

Personalize learning

Connection to prior learning

Explicit connections made between ideas

Links to other subjects

Less is more

Engagement focus

Significant student contribution to lesson

## Appendix 7: Coded strategies

*A Allow students to investigate*

*E Allow students to make errors*

*R Ask students for reasoning*

CQ Closed questions

G Definitions given used

CD Definitions presented explained

The preceding two strategies were combined to create

CD Definitions presented explained

*D Direct instructions*

*T Follow textbook-worksheets*

U Calculator use

C Identified content at the start of the lesson

*L Identify likely errors or confusion*

SG Lesson starter question-game

SQ Lesson starter quiz

The preceding two strategies were combined to create

S Lesson starter

*M Manipulatives issued to students*

MO Model procedure or process

N Notes provided by teacher

CQ Choice of questions

DP Self assessment or peer

assessment

W Students make their own notes

WI	Students work individually
WO	<i>Students work with others</i>
TA	Teacher assessment
P	Use of practice in class
I	Whole class instruction
PL	Personalize learning
CL	Connection to prior learning
EC	Explicit connections made between ideas
LS	Links to other subjects
LM	Less is more-Time for elaboration
EF	Engagement focus
SC	Significant student contribution to lesson

## Appendix 8: Marie's: LESSON 1\_ Transcript and initial coding of strategies

.....Would like you to try this on the board. He was trying to do a calculation.  $14 \times 82$  But then he wrote down wrong he wrote down  $41 \times 28$  so the questions here are. What answer did he get. What answer should he have got? Are there any other similar calculation that he would have got write even although he had wrote down those digits wrong? You can use that paper or you can use your show me on your ipads or you can use my board **SQ, D,LM, CL,R,A**

What did he get? What should he have got and are there some numbers where that would actually work? Where he would get the right answer even although he got wrote the digits the wrong way round? **R,**

Can you show your methodology? **R**

So tell me what is going on? So if you think about it ..  $14 \times 2$  is 28 that's correct, that's correct so far. How do you normally do the next step? **EF,**

Just looking this way for a moment thanks everyone. Most people when they saw this a multiplication straight away put the numbers in columns and decided to use this method. And I think that has got a lot to do with that's how you got taught in primary school so that's the method that stays with you, is that right? You tend not to think about other of ways that you could possibly do this but we will start with that. Here is something I saw a couple of people doing  $4 \times 2$  is 8 and then  $1 \times 8$  is 8 so that's 88 but I know that there is something else because in this method there is a piece of adding isn't there? Then people got a bit stuck because they couldn't remember what to do so that is why I am not a huge fan of this method because that is such an easy mistake to make. **I,R, E,CL, L**



We will look at how we should have done it.  $4 \times 2$  is 8 with this method the next step would then be to say we know that  $1 \times 2$  is 2 now just think about that for a minute. Imagine that the 80 wasn't there at all and this was just  $14 \times 2$ ,  $14 \times 2$  is 28 so that's ok, that's fine but the next part is to try and work out where the 80 fits in. We have done  $14 \times 2$  so we need to work out  $14 \times$  the 80. When you are working through this method you tend to forget that this is really 80 and you just think about it being an 8 so how do you compensate for that?

How do you make sure that it is 80? **I, MO, R**

....Do you know what  $8 \times 4$  is? 32 that's right is this how you were taught to do it? **CL,**

...Can you talk me through this?

Do you normally put the three up there or put in difference places? You put it next to the 1 that's fine. What I want you to think about here is how does that method work?

You put the numbers there you do this you do that you add this that you carry that there done answer and unless you make a mistake its fine. Let's think about what we are actually doing.

This 28 here was  $14 \times 2$  and what is this 1120.....  $14 \times 80$  very good and then we added them together. See if you can work out what each line of the calculation represents see if you can

work out what each line of your calculation represents. **I, R, A, MO**

.....Try and see if you can do the second one like I did the first one how do you work out what each line represents. **P**

Let's have a look at the second one then. He wrote it down wrong. He wrote  $41 \times 28$  and if you use a similar method to before you would start by saying, forget about the 20 for a minute and say  $41 \times 8$  Is that how you did it? (teacher does calculation) Is that right? What does that line represent?

What have I worked out? Before we started with  $14 \times 2$ ... This is  $8 \times 41$  or  $41 \times 8$  So what's the second line going to be when you have done that? ....What did you get?

Or you might have written  $20 \times 41$  and so is that...What did you get for your answer to that line ....?.....Did you add them together? What did you get? .....

Is that right? Some people got different answers ....I got .....so what has gone wrong there? If you got different answers you had better figure what's gone wrong, should they be the same or should they be different? ....the same).

So if you got different answers show the person next to you see if you figure out which step did you get wrong? **MO,E,D**

Let me write this one over here (on whiteboard) You did  $41 \times 18$  Did you figure out where you went wrong? Have you all figured out where you went wrong if you got different answers? So does that always work? Is it ok to switch the digits round? Try another couple of two digit numbers your choice and if you mix up the digits do you still get the same answers each time?

**A, R,D,MO**

You can use calculators now for this I actually want you to try and explore. Do you always get the same answer when you swap the digits or is it something special about those numbers?

....I got the same **A, U**

...

... represents student response.

## Appendix 9: Unit of Work Grid -Year Seven Mathematics

### Content Strand: Number and Algebra

80 minutes	80 minutes	80 minutes
<p><b>Lesson 1</b>  <b>Review of Order of Operations.</b>            BIDMAS (brackets, indices, division, multiplication, addition and subtraction).  <b>Review of Rules of Arithmetic.</b>            associative law:  <math>(a+b) + c = a + (b + c)</math>            commutative law:  <math>a+b = b+a</math> : <math>ab = ba</math>            distributive law: <math>a \times (b + c) = ab + ac</math>            mental and written computation            Worksheet from Understanding Mathematics (pages 347-354)            Number Charts Lesson 84 (Maths 300)</p> <p><u>Research note 1:</u>            A model of algebra as a progression of arithmetic, is not without merit. It can be argued that students need to have a good understanding of and fluency in using the rules of arithmetic if they are to deal with the abstractions of algebra.</p> <p><u>Research note 2:</u>            Balance is needed between procedural fluency and conceptual understanding. (NTCM 2000, P.35, Resek and Kirshner 2006, P 539.)</p> <p><u>Australian Curriculum:</u>  <i>Apply the associative law, commutative law and distributive law with numbers, mentally. Use the order of operations rules</i>            (The Australian curriculum, 2012)</p>	<p><b>Lesson 2</b>  <b>Review of Integers</b>            Positive and Negative Integers            The number line            Compare integers            Four operations using integers</p> <p>Booklet on 4 operations and problems.</p> <p><u>Research note 1:</u>            Britt and Irwin in the NZ Numeracy Project of 2005 found that when students could represent number operational strategies successfully they were more likely to be able to generalize and therefore to illustrate algebraic thinking strategies.</p> <p><u>Research note 2:</u>            Understanding integers is important and the ability to use them fluently in calculations for learning algebra Wu, (2001)</p> <p><u>Australian Curriculum:</u>  <i>Apply the associative law, commutative law and distributive law with numbers, mentally</i>  <i>Use the order of operations rules</i>            (The Australian curriculum, 2012)</p>	<p><b>Lesson 3</b>  <b>Review of Order of Operations</b>  <b>Review of the Rules of Arithmetic</b>  <b>Review of Integers</b>            Positive and Negative Integers            The number line            Compare integers            Four operations using integers</p> <p>Booklet on 4 operations and problems.</p> <p>Mini class test 20 questions.</p> <p><u>Research note:</u>            Sousa (2008), has suggested that moving students from the use of concrete materials to abstract thinking is in essence a movement from number to algebra.</p> <p><u>Australian Curriculum:</u>  <i>Apply the associative law, commutative law and distributive law with numbers, mentally</i>  <i>Use the order of operations rules.</i>            (The Australian curriculum, 2012)</p>

80 minutes	80 minutes	80 minutes
<p><b>Lesson 4</b>  <b>Introduction to the concept of algebraic variable.</b></p> <p>1089 Lesson 120 (Maths 300)  Sadler Understanding Mathematics worksheet p119-126.</p> <p><u>Research note:</u>  Transition to symbolization is complex, difficult, and facilitating the transition to symbolization is not well understood. (Knuth et al. 2006, pp297-312.). Students' will benefit from being taught algebraic techniques and strategies, if they are first and foremost comfortable with the notion of a variable.</p> <p><u>Australian Curriculum:</u>  Apply the associative law, commutative law and distributive law with variables. Generalise these processes using variables.  Use the order of operations rules (The Australian curriculum, 2012)</p>	<p><b>Lesson 5</b>  <b>Using variables to solve problems.</b></p> <p>Addition Totals Lesson 18 (Maths 300)</p> <p><u>Research note:</u>  According to Kaput algebra is more than a continually developing body of knowledge but also a way of thinking, reasoning, talking and doing. (Kaput; 2008). Viewed from this perspective algebra includes both content and process. The content has its roots in cultural development. As societies needs have become more complex then too has the need to solve more and more abstract problems. Knowledge of the rules and conventions used in algebra is key to using algebraic techniques successfully to solve problems.</p> <p><u>Australian Curriculum:</u>  Apply the associative law, commutative law and distributive law with variables. Generalise these processes using variables.  Use the order of operations rules (The Australian curriculum, 2012)</p>	<p><b>Lesson 6</b>  <b>Using variables to solve problems.</b></p> <p>4-Arm Shapes Lesson 40 (Maths 300)</p> <p><u>Research note :</u>  The work of Vygotsy (1978), a contemporary of Piaget, laid a great emphasis in the place of collaboration with others to construct knowledge.</p> <p><u>Australian Curriculum:</u>  Apply the associative law, commutative law and distributive law with variables. Generalise these processes using variables.  Use the order of operations rules (The Australian curriculum, 2012)</p>

80 minutes	80 minutes	80 minutes
<b>Lesson 7</b> <b>Algebraic terms and expressions</b> <b>Order of Operations</b> <b>Rules of Arithmetic</b> <b>Using Integers</b>	<b>Lesson 8</b> <b>Algebraic terms and expressions</b> <b>Order of Operations</b> <b>Rules of Arithmetic</b> <b>Using Integers</b>	<b>Lesson 9</b> <b>Collecting like terms</b> <b>addition and subtraction</b> <b>operations using algebraic</b> <b>terms</b> <b>Order of Operations</b> <b>Rules of Arithmetic</b> <b>Using Integers</b>
Algebra Charts Lesson 160 (Maths 300)	Algebra Charts Lesson 160 (Maths 300)	
<u>Research note:</u> Research undertaken by Radford (2000) and others supports the notion that learning to think algebraically requires social immersion into activities, which use algebraic symbols, signs and actions.	<u>Research note:</u> The work of Nothfield, Gunstone, & Erickson, (1996) support collaboration as the basis of teaching and of learning, noting the central role the teacher plays in student learning.	
<u>Australian Curriculum:</u> Apply the associative law, commutative law and distributive law with variables. Generalise these processes using variables. Use the order of operations rules (The Australian curriculum, 2012)	<u>Australian Curriculum:</u> Apply the associative law, commutative law and distributive law with variables. Generalise these processes using variables. Use the order of operations rules (The Australian curriculum, 2012)	<u>Australian Curriculum:</u> Apply the associative law, commutative law and distributive law with variables. Generalise these processes using variables. Use the order of operations rules (The Australian curriculum, 2012)

80 minutes	80 minutes	80 minutes
<b>Lesson 10</b> <b>Collecting like terms</b> <b>Addition and subtraction operations using algebraic terms</b> <b>Order of Operations</b> <b>Rules of Arithmetic</b> <b>Using Integers</b>	<b>Lesson 11</b> <b>Operations involving Multiplication and Division using algebraic terms</b> <b>Order of Operations</b> <b>Rules of Arithmetic</b> <b>Using Integers</b>	<b>Lesson 12</b> <b>Operations involving Multiplication and Division using algebraic terms</b> <b>Order of Operations</b> <b>Rules of Arithmetic</b> <b>Using Integers</b>
<p><u>Research note:</u> Sfard (2000a) proposed that students should not be expected to discover the meta rules of algebra. These rules and the appropriate language need to be taught and modelled by the teacher. To give too much weight to profound constructivist principles can rob the students of the powerful role the teacher can play in the students' learning.</p> <p><u>Australian Curriculum:</u> Apply the associative law, commutative law and distributive law with variables. Generalise these processes using variables. Use the order of operations rules (The Australian curriculum, 2012)</p>	<p><u>Research note:</u> Ma (1999) argued that "to promote mathematical understanding, it is necessary that teachers help to make connections between manipulatives and mathematical ideas explicit" (p. 6).</p> <p><u>Australian Curriculum:</u> Apply the associative law, commutative law and distributive law with variables. Generalise these processes using variables. Use the order of operations rules (The Australian curriculum, 2012)</p>	<p><u>Research note:</u> Brousseau (1997) proposes the notion of "didactical transposition". That is that teaching involves presenting an idea in more than one context and in doing so allowing the learner to personalize the experience.</p> <p><u>Australian Curriculum:</u> Apply the associative law, commutative law and distributive law with variables. Generalise these processes using variables. Use the order of operations rules. (The Australian curriculum, 2012)</p>

80 minutes	80 minutes	80 minutes
<p><b>Lesson 13</b>  <b>Finding a rule to describe a relationship.</b>  <b>Using words and symbols to describe the rule</b>  Text workbook pages 140-141 Q 1-6.  Evaluate expressions Sadler Understanding Mathematics worksheet pages 357-365</p> <p><u>Research note:</u>  Algebra is about thinking and expressing this thinking in a particular way.  “learning mathematics is easier when it makes sense and is meaningful to the learner” (Sousa, 2008, p215)</p> <p><u>Australian Curriculum:</u>  Generalise these processes using variables.  Create linear equations  (The Australian curriculum, 2012)</p>	<p><b>Lesson 14</b>  <b>Finding a rule to describe a relationship.</b>  <b>Using words and symbols to describe the rule</b>  Text workbook P175 -177 Investigation and activity.  Sadler Understanding Mathematics worksheet pp366-374.</p> <p><u>Research note:</u>  Students need to acquire a sense of the big ideas of algebra.  Researchers have found that algebra students generally lack a sense of the big structural ideas and, therefore, come to believe algebra at its core is memorizing rules and procedures (Kieran 1992).</p> <p><u>Australian Curriculum:</u>  Generalise these processes using variables.  Create linear equations  (The Australian curriculum, 2012)</p>	<p><b>Lesson 15</b>  <b>Create, solve and interpret linear equations, including those using realistic contexts.</b>  Dog problem ppt.+ worksheet  Text p 233 Investigation  Text Workbook Ch. 9 Ex 9.5</p> <p><u>Research note :</u>  Researchers found that students in middle school did not exhibit a relational understanding of the equal sign unless they had contextual support. It was apparent that, students did not interpret the equal sign as a relational symbol of equivalence in general, but they were able to interpret the equal sign as a relational sign in the context of an equation with operations on both sides of the equal sign (McNeil &amp; Alibali, 2005).</p> <p><u>Australian Curriculum:</u>  Generalise these processes using variables.  Create linear equations  Solve linear equations  Interpret linear equations  [ including those using realistic contexts using algebraic and graphical techniques.]  (The Australian curriculum, 2012)</p>

80 minutes	80 minutes	80 minutes
<p><b>Lesson 16</b>  <b>Create, solve and interpret linear equations, including those using realistic contexts.</b></p> <p>Text p 234 Ex 9.6 Q 6-16  Solving problems with equations</p> <p><i>Australian Curriculum:  Generalise these processes using variables.  Create linear equations  Solve linear equations  Interpret linear equations  [ including those using realistic contexts using algebraic and graphical techniques.]</i></p> <p>(The Australian curriculum, 2012)</p>	<p><b>Lesson 17</b>  <b>The Cartesian Plane</b>  Maths in Action Text pages 182-183.  Text Workbook P 142-145  Grid references</p> <p><i>Australian Curriculum:  Create linear equations  Solve linear equations  Interpret linear equations  [ including those using realistic contexts using algebraic and graphical techniques.]  Plot graphs of linear functions  Use graphs of linear functions to find solutions of equations including using ICT.</i></p> <p>(The Australian curriculum, 2012)</p>	<p><b>Lesson 18</b>  <b>The Cartesian Plane</b>  Text Workbook P 146-149  Grid references</p> <p><i>Australian Curriculum:  Create linear equations  Solve linear equations  Interpret linear equations  [ including those using realistic contexts using algebraic and graphical techniques.]  Plot graphs of linear functions  Use graphs of linear functions to find solutions of equations including using ICT.</i></p> <p>(The Australian curriculum, 2012)</p>



80 minutes	80 minutes	80 minutes
<p><b>Lesson 19</b>  <b>Plot graphs of linear functions and use these to find solutions of equations including using ICT</b>  Algebra Walk lesson 22 (Maths 300)  Sadler Understanding Mathematics worksheet p375-384.</p> <p><i>Australian Curriculum:</i>  Generalise these processes using variables.  Create linear equations  Solve linear equations  Interpret linear equations  [ including those using realistic contexts using algebraic and graphical techniques.]  Plot graphs of linear functions  Use graphs of linear functions to find solutions of equations including using ICT.</p> <p>(The Australian curriculum, 2012)</p>	<p><b>Lesson 20</b>  <b>Plot graphs of linear functions and use these to find solutions of equations including using ICT</b>  Algebra Walk Lesson 22 (Maths 300)</p> <p><i>Australian Curriculum:</i>  Generalise these processes using variables.  Create linear equations  Solve linear equations  Interpret linear equations  [ including those using realistic contexts using algebraic and graphical techniques.]  Plot graphs of linear functions  Use graphs of linear functions to find solutions of equations including using ICT.</p> <p>(The Australian curriculum, 2012)</p>	<p><b>Lesson 21</b>  <b>Plot graphs of linear functions and use these to find solutions of equations including using ICT</b></p> <p><i>Australian Curriculum:</i>  Generalise these processes using variables.  Create linear equations  Solve linear equations  Interpret linear equations  [ including those using realistic contexts using algebraic and graphical techniques.]  Plot graphs of linear functions  Use graphs of linear functions to find solutions of equations including using ICT.</p> <p>(The Australian curriculum, 2012)</p>

80 minutes	80 minutes	80 minutes
<b>Lesson 22</b> Famous Mathematicians Lesson 124 (Maths 300) <i>Revision</i>	<b>Lesson 23</b> Famous Mathematicians Lesson 124 (Maths 300) <i>Revision</i>	<b>Lesson 24</b> <b>Class test</b>

## Appendix 10: Lesson Notes

### Lesson Three

**Materials Required: A box of six sided dice.**

Aims	Fluency in the use of the laws of arithmetic. Fluency in the use of order of operations rules in integers.	Est. Time
Objectives	<ol style="list-style-type: none"> <li>1. Review of Order of Operations</li> <li>2. Review of the Rules of Arithmetic</li> <li>3. Review of Integers <ul style="list-style-type: none"> <li>Positive and Negative Integers</li> <li>The number line</li> <li>Compare integers</li> <li>Four operations using integers</li> </ul> </li> </ol> <p>(write these on whiteboard at the start of the lesson)</p>	
Intro (Motivate, Interest)	<p>Teacher throws four dice. A red, a blue, a white and a yellow. The four resultant numbers are then put on large pieces of card and stuck to the whiteboard. Using these four digits I could create large or small numbers. What is the largest number I can make? What is the smallest number I can make? Write answers on the board. What is the difference between the largest number and the smallest number? I am now going to ask you 10 questions using only these four digits. All calculations have to be done in your head. (you can use pen and paper no calculators) It will be a quick quiz so time will be limited. In your maths work book turn to the back pages put the heading quick quiz and number 1 to 10. Remember to date your work. This task is for you alone so there should be no talking. Just do what you can. I would like you to write out the question each time before you put down your answer. Any questions? Quick quiz questions-</p> <ol style="list-style-type: none"> <li>1. <math>52+34</math></li> <li>2. <math>23+54</math></li> <li>3. <math>534 \times 2</math></li> <li>4. <math>24 \div 3</math></li> <li>5. <math>3(5+2)</math></li> <li>6. <math>4+5 \times 2 + 3</math></li> <li>7. <math>4 \div 2 - 5</math></li> <li>8. <math>52-43</math></li> <li>9. <math>5 \times 4 \div 2 + 3</math></li> <li>10. <math>2-4-3 \times 5</math></li> </ol> <p>Mark work</p>	15 minutes
Body of	A good sense of number is very useful at all stages of life. Can anyone	20 minutes

Lesson	<p>tell me where you use number in your life?</p> <p>What numbers do you think are important to your parents, grandparents?</p> <p>In Year Seven our focus needs to be on ensuring that you can use the four number operations fluently and not just with whole numbers but with negative numbers as well.</p> <p>In the previous two lessons we have covered a review of each separately today we are putting it all together.</p> <p>So much of the future learning in algebra is built on your ability to choose and use number operations well. Algebra extends our ability to use number and ultimately to generalize. We will look at what generalizing means later.</p> <p>Lets look at your answers for the quick quiz questions. Numbers 6,7,9 and 10 may have caused some problems for some of you.</p> <p>Why would I pick these questions and why would I expect there to be errors?</p> <p>Project on the screen or show on the IWB the order of operation rules. What happens if you do not stick to these rules of operations with number?</p> <p>Go through rules with the class.</p> <p>Copy this note into your maths Journal. You need to spend time learning these rules.</p> <p>What other rules have we been looking at recently?</p> <p>Project on the screen or show on the IWB the rules (Laws) of arithmetic.</p> <p>Go through rules with the class.</p> <p>Copy this note into your maths Journal. You need to spend time learning these rules.</p> <p>The third objective in our lesson today is to review integers.</p> <p>Project on the screen or show on the IWB the extended number line and operations with integers.</p> <p>Go through dealing with integers in calculations.</p> <p>Copy this note into your maths Journal. You need to spend time learning these rules.</p>	
Rehearsal/ Practice	Booklet on 4 operations and problems.	15 minutes
Closure	Mini class test 20 questions (teacher collects to mark)	15 minutes
Game	<p>Give out 3 dice to pairs of students</p> <p>Instructions for the game. Player 1 throw the dice then write down the answer to a calculation using the 3 numbers.</p> <p>Player 2 has to guess what operations you did to get your answer.</p> <p>They need to get it write within 2 guesses or it is player 1's turn again.</p> <p>5 points each time you succeed.</p>	10 minutes
<u>Research note:</u>	Sousa (2008), has suggested that moving students from the use of concrete materials to abstract thinking is in essence a movement from number to algebra.	
	<p><i>Apply the associative law, commutative law and distributive law with numbers, mentally</i></p> <p><i>Use the order of operations rules.</i></p> <p><i>(The Australian Curriculum, 2012)</i></p>	

## Lesson Notes (Lesson 16)

**Materials Required: Copies of investigation. Computers or ipads**

**Several stick it note pads.**

Aims	Create, solve and interpret linear equations, including those using realistic contexts.	Est. Time
Objectives	<ol style="list-style-type: none"><li>1. Investigate different methods of solving linear equations.</li><li>2. Use a computer spreadsheet to investigate change in variables</li></ol>	
Introduction (Motivate, Interest)	<p>Investigation-What does this word mean? It is my experience that in the maths classroom the use of this word to describe a task will often make students especially senior students very apprehensive. This is because it is an open task. It is not like doing an exercise from the textbook where you practice a skill. This is about using what you know about mathematics and using this to describe and experiment. Your thinking needs to be made clear so there will be words used. You need to explain why you chose to do something and what results you found. As you are in year 7 this investigation will have guidelines. Please ask if you need help.</p> <p>For this lesson I want you to investigate solving equations. You will work in groups of two and at anytime during the lesson you may be asked to report your finding to the class. Therefore it is important to write down what you find as you investigate. How you record your findings will be up to you. The order you decide to tackle the investigation is also up to you. You will need to use your computer and or ipad for part of the investigation.</p> <p>Hand out investigation to students.</p> <p>Are there any questions about what you have to do? I will be coming around to see each group in turn. If you have a pressing question use the stick notes pad and put your question on the whiteboard remember to put at least one persons name on it so I know who to come to.</p>	15 minutes
Body of Lesson	<p>The teacher ensures all groups have begun and that they are clear on how to proceed by walking around the classroom and talking to students in their small groups.</p> <p>The teacher moves to his/her desk and waits to consult for say 5-10 minutes this will depend on how well the class are handling the investigation.</p>	25 minutes
Rehearsal/	<p>Reporting back period. Each pair of students must pair with another and take turns in reporting back on their findings. Students are advised to look for similarities or any</p>	10 minutes

Practice	differences. They make the judgement on the findings.	
Closure	The teacher brings all groups together to collect the findings, which are displayed to the whole class.	15 minutes
Game	Stand up numbers game	10 minutes
<u>Research note:</u>	<u>Research note:</u> Students need to acquire a sense of the big ideas of algebra. Researchers have found that algebra students generally lack a sense of the big structural ideas and, therefore, come to believe algebra at its core is memorizing rules and procedures (Kieran 1992).	
	<i>Generalise these processes using variables. Create linear equations Solve linear equations Interpret linear equations [ including those using realistic contexts using algebraic and graphical techniques.] (The Australian Curriculum, 2012)</i>	

#### Lesson 6: Cover Note

Materials Required: See lesson notes attached

Aim	Using variables to solve problems
	This lesson is taken from the Maths 300 series of lessons: Website <a href="http://www.maths300.esa.edu.au">www.maths300.esa.edu.au</a> Garden beds Lesson 16
<u>Research note:</u>	The work of Vygotsky (1978), a contemporary of Piaget, laid a great emphasis in the place of collaboration with others to construct knowledge.
	<i>Apply the associative law, commutative law and distributive law with variables. Generalise these processes using variables. Use the Order of operations rules (The Australian Curriculum 2012)</i>

Appendix 11: Examples of students' notes from Marie's class

we have

to get

all these

budgets and put them in

order so the relationship

is the same.

$2(5+6) = 10+12$

$-3(2a-5) = -6a-15$

$8x-8$

$3(2a-5)$

$a-4$

$2(x+6)$

$(1+2)(3a+4)$

$10x-20$

$6a+8$

$(a+4)$

$2a+6$

$8(x-1)$

$10(x+2)$

$18a-24b$

$6(3a-4b)$

$2x+12$

## Expression < Equation

11 4 12

- Simplifying algebra expressions
- Expanding brackets
- Equations
- Inverse operations
- Solving equation

### Algebraic Terms

coefficient (number)

variable (letter/symbols)

Terms:

$4x$   $30$   $7a$   $\frac{1}{2}b$   $\frac{m}{4}$  or  $\frac{1}{4}m$

$2ac$   $5d$   $265 = \text{coefficient}$   
 $26d = \text{variable}$

$$d \div 3 = \frac{d}{3} = \frac{1}{3}d \quad \frac{5x}{2} = \frac{5}{2}x$$

$$5x \div 2 = 2.5x = \frac{5x}{2}$$

Examples

$(9d)$  coefficient = 9



Writing Expression in Different ways

Expand  $3(2x+4)$

$$3(2x+4) = 3(2x) + 3(4) = 6x + 12.$$

$$4(3y+2) = 12y + 8, \quad \text{expanding}$$

$$10x + 12 = 2(5x+6)$$